Remarks on granularity of PARIS

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Doppler broadening is the spread in measured energy caused by γ rays from moving nuclei which are observed using a stationary detector. The effect gets worse when the velocity of the nucleus increases and as the detector size (opening angle seen by nucleus) is increased. Doppler broadening can be corrected (equation 1), but to do so the angle between the origin of the γ rays and the detection point should be known.

Equation for relativistic Doppler shift effect:

$$E = \frac{E_0}{\gamma} \cdot \frac{1}{1 - \beta \cdot \cos\Theta} \tag{1}$$

In addition there is always an uncertainty in the velocity value of the nucleus ($\Delta\beta$) and an uncertainty about the direction of source ($\Delta\Theta_r$). This is neglected in this paper.

Uncertainty which is connected to doppler broadening [1] is related to opening angle of the detector $\Delta\Theta$

$$\Delta E_{\Delta\Theta} = \int_{\Theta-\Delta\Theta}^{\Theta+\Delta\Theta} \frac{\delta E}{\delta\Theta} d\Theta = \frac{E_0}{\gamma} \cdot \frac{2 \cdot \beta \sin(\Theta) \sin(\Delta\Theta)}{1 - 2 \cdot \beta \cos(Theta) \cos(\Delta\Theta) + \beta^2 \cos(\Theta - \Delta\Theta) \cos(\Theta + \Delta\Theta)} \tag{2}$$

To check what value of opening angle will lead to constant $\frac{\Delta E}{E}$, for various Θ, function of $f(\Theta) = \Delta\Theta$ obtained from equation 2 by some approximation, was used:

$$\Delta\Theta = \arcsin\left(\frac{k \cdot \gamma \cdot (\beta \cdot \cos(\Theta) - 1)^2}{2 \cdot \beta \sin(\Theta)}\right) \tag{3}$$

where $\frac{E}{E_0} = k$ was constant in full range of Θ.

On figures 1 and 2 there are curves for $\beta = 10\%$ up to 60% for respectively k= 3 % and 1.5 %. Minimal values of $\Delta\theta$ for each of them are in table 1. In the same table there are solid angles Ω covered by one detector which has mimimal $\Delta\Theta$ opening angle - taken from equation 4. Number of detectors, for different values of β, obtained by dividing $4\pi/\Omega$ are presemted in table 1, and plot in logarithmic scale on figure 3.

The conclusion is that realistic number of detectors $\approx$ 800 would be sufficient to cover reactions up to $\beta = 20\%$. And having inner radius of 20 cm will require use of detectors with 1" diameter.

[1] taken from PARIS raport "A general study on segmentation" by O. Stezowsk, Ch. Schmitt

Table 1: Number of detectors which covers 4π when detector opening angle is equal to minimal $\Delta\theta$, for different beta and k

β [%]	k=$\frac{\Delta E}{E}$ [%]	minimal $\Delta\Theta$ [degree]	Ω	number of detectors
10 %	1.5 %	4.2	0.0169	743
20 %	1.5%	2	0.00383	3283
30 %	1.5 %	1.25	0.00149	8404
40 %	1.5 %	0.84	0.000675	18610
50 %	1.5 %	0.59	0.000333	37723
60 %	1.5 %	0.4	0.000153	82070
10 %	3 %	8.5	0.0690	182
20 %	3 %	4	0.0153	821
30 %	3 %	2.5	0.00598	2101
40 %	3 %	1.69	0.00273	4598
50 %	3 %	1.17	0.00131	9593
60 %	3 %	0.81	0.000628	20014

$$\Omega = \frac{4\pi}{2}(1 - \sqrt{1 - \sin^2(\Delta\Theta)}) \tag{4}$$

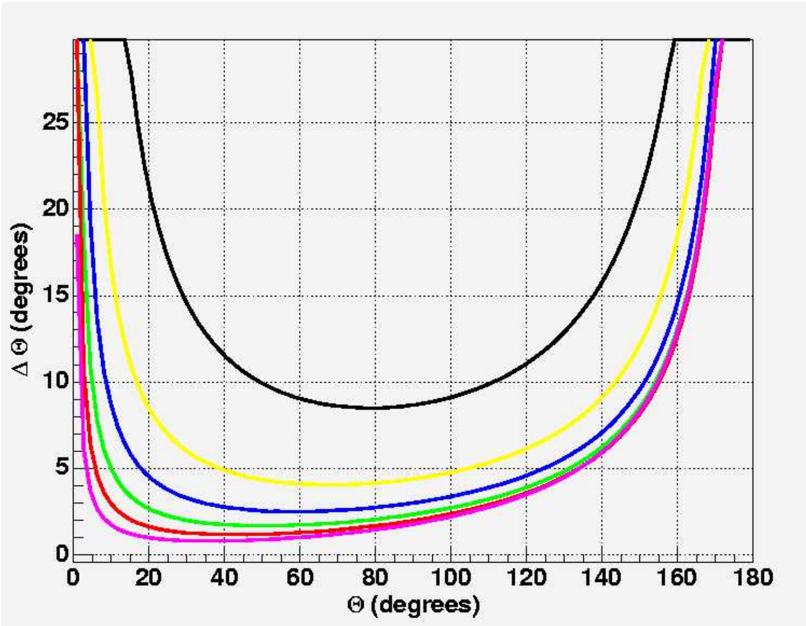

Figure 1: Equation 3 ploted for beta=10% up to 60% and energy smearing k=3%. Black - beta 10%, yellow - 20%, blue - 30%, green - 40%, red - 50%, pink - 60%.

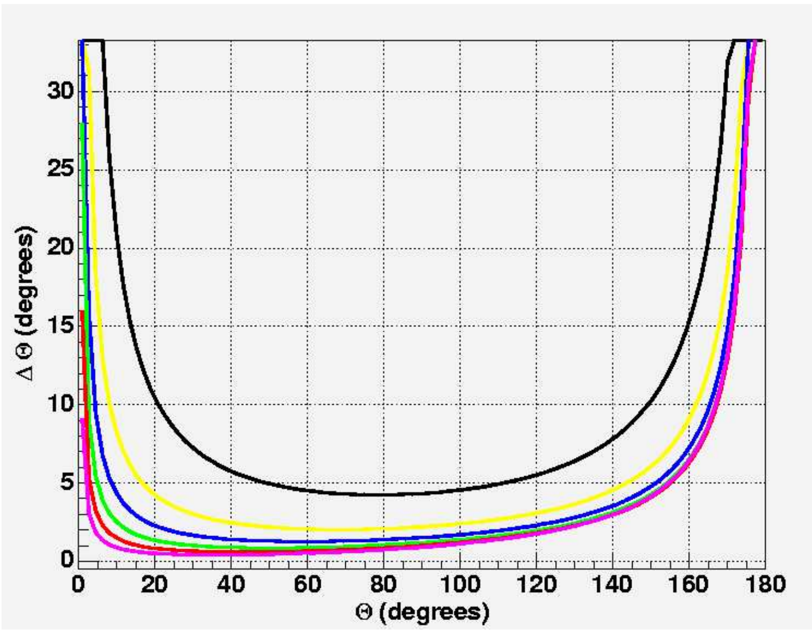

Figure 2: Equation 3 ploted for beta=10% up to 60 % and energy smearing k=1.5%. Black - beta 10%, yellow - 20%, blue - 30%, green - 40%, red - 50%, pink - 60%.

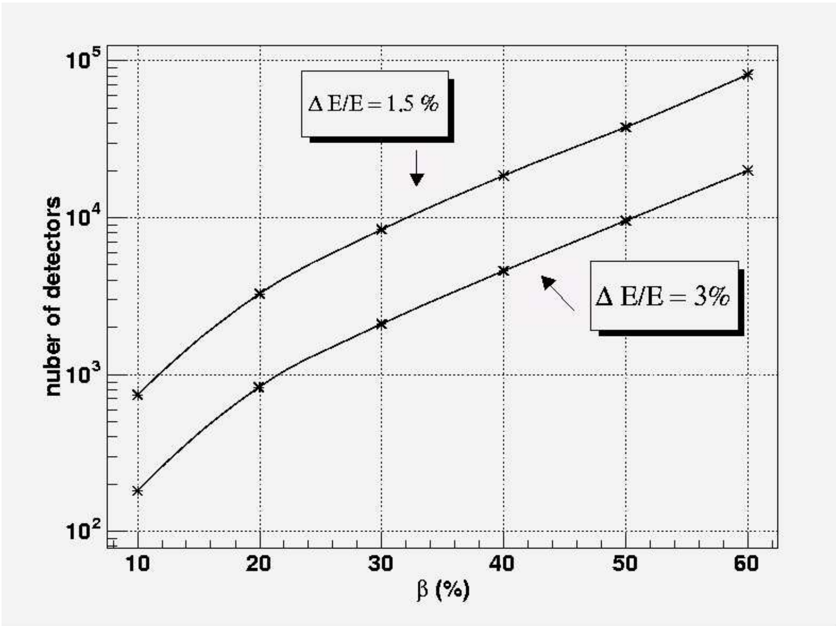

Figure 3: Number of detectors which should be used to cover 4π when $\Delta E/E$ is not worse then 3% or 1.5%