PHOTON ARRAY FOR STUDIES WITH RADIOACTIVE ION AND STABLE BEAMS

A general study on segmentation

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Abstract

In this report, several effects that could constrain the segmentation of the Paris array are studied. The determination of the size(s) of the cells which will composed the detector relies on many factors. Geant4 simulations help to optimize the segmentation by showing how an incident $\gamma-$ray (or a more complex source of $\gamma-$rays) is absorbed, depending on the energy: this will be treated later on. Rather, the present paper focuses on general effects, treated without any simulations, that could affect the performances of the array, namely Doppler shift, Doppler broadening and pile-up.

Physics cases's requirements

In the Krakow meeting (14-15 May 2007), the following table (Table1) has been established to summarize, for the different physics cases, the required performances of the Paris detector.

	dynamics	ΔE	$\Delta\theta$	counting rates	$\Delta\Omega$	ΔT
Jacobi transitions	$100\,keV \rightarrow 30\,MeV$	5 MeV	$\Delta L = 5\hbar$		4π	
Differential technique for GDR	$100\,keV \rightarrow 30\,MeV$	5 MeV	$\Delta L = 5\hbar$		4π	
GDR properties	$100\,keV \rightarrow 30\,MeV$	5 MeV	$\Delta L = 5\hbar$		4π	
isospin mixing	$100\,keV \rightarrow 30\,MeV$	5 MeV	$\Delta L = 5\hbar$		4π	
multifragmentation					2π	
reaction	$100\,keV \rightarrow 30\,MeV$	5 MeV	$\Delta L = 5\hbar$	$10^4 - 10^5$	4π	1ns
radiative capture	$100\,keV \rightarrow 40\,MeV$	2-3%			4π	
fragmentation		3%			4π	
astrophysics					4π	

Table 1: general constraints for the different physics cases

Remarks:

- requirements related to the segmentation of the array are in the $\Delta\theta$ column
- the hole for the beam pipe is not substracted in the $\Delta\Omega$ coverage column

Doppler shift

Most of the study presented here relies on the non-relativistic Doppler shift formula which is given in Eq1. with the conventions illustrated in Fig1.

A $\gamma-$ray emitted by a moving source along the horizontal axis is detected at the angle θ with an energy (E^d) different from the emitted one (E^0):

$$E^d = E^0(1 + \beta\cos(\theta)) \quad Eq1.$$

where β corresponds to the recoil velocity (in c units). Because the used function works for non-relativistic moving sources, **it is important to define the upper limit for β** imposed by all the physics cases to be sure the performances will be reached in all cases. In particular, the Lorentz transformation boosts the laboratory energy in such a way it breaks the symmetry $\theta \rightarrow \theta + 90^o$.
If we considere the value $\beta_{max} = 0.1$ (3 centimeters per second or an energy of 4.7 AMeV) as the upper limit for the recoil velocity, the Doppler shift is maximum at 0 and 180 degrees and correspond to a shift of 10%. Thus a 1 MeV $\gamma-$ray will be detected with an energy of 1100 keV.

The relativistic Doppler effect is given by Eq2:

$$E^d = \frac{E^0}{\gamma}\frac{1}{1-\beta\cos(\theta)}, \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}} \quad Eq2.$$

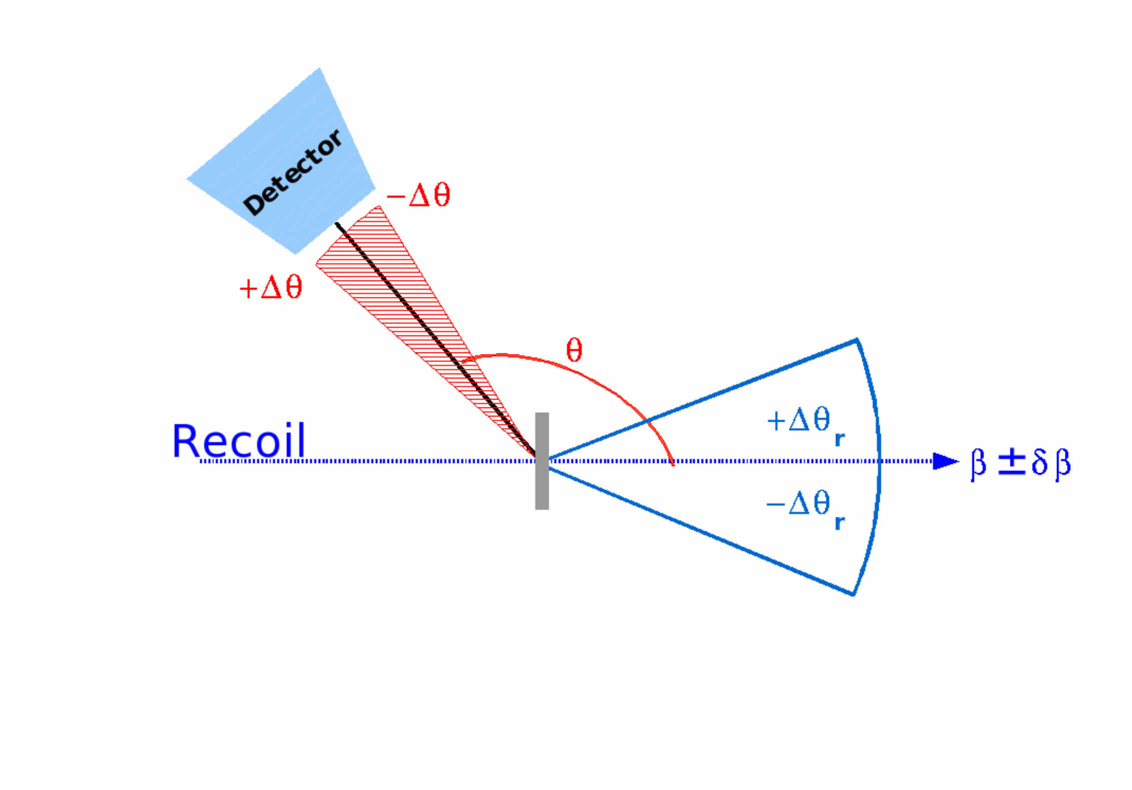

Figure 1: Conventions for the parameters involved

To illustrate the differences between the two formula, several curves have been drawn in Fig2. The energy of a 10 MeV $\gamma-$ray (in center of mass) is detected as a function of the angle (from 0 to 180 degrees) and for three different recoil velocities (3, 10 and 80 %). In blue is represented the classical Doppler effect while the relativistic one is represented in red. In the left panel, only the red curve could be seen because the two formula match perfectly. Even for a recoil velocity around 10%, the difference between the two shifts is negligible (panel in the middle). However, for the highest recoil velocity (right panel), the clear effect of the Lorentz boost is seen with an important breaking of the symmetry.

In conclusion, as soon as the PARIS physics cases concern recoil velocity lower than 10% of c, the classical formula is used to estimate, in this report, what could be the characteristics of the PARIS geometry.

Doppler broadening

The energy is detected with uncertainties (ΔE_{tot}) coming from the intrinsic resolution of the detector (ΔE_{int}) and from the Doppler correction (ΔE_{dop}) applied.

$$\Delta E_{tot} = \sqrt{(\Delta E_{int})^2 + (\Delta E_{dop})^2}$$

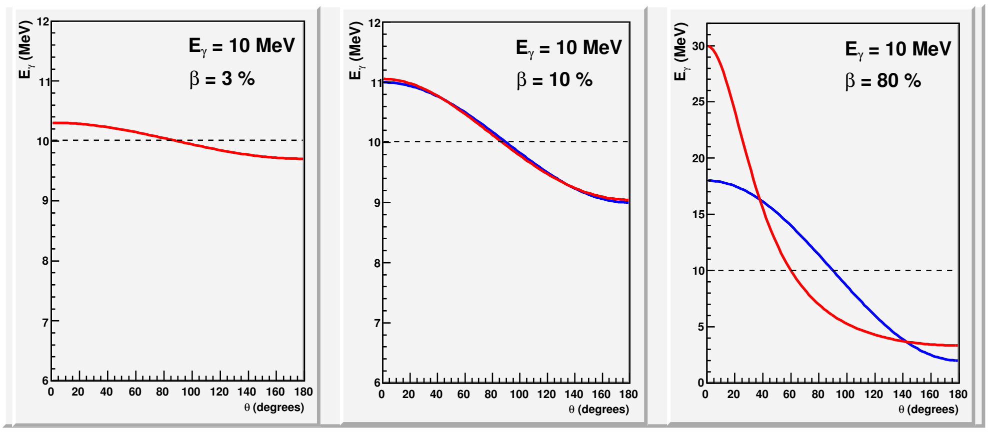

Figure 2: Doppler shift as a function of the θ angle for a 10 MeV γ-ray for three different recoil velocities. In blue, the classical formula is applied while in red the relativistic one is applied.

For the intrinsic resolution, LaBr3 is choosen for the study. According to previous studies[1], it is given by:

$$\Delta E_{int} = 76.3\sqrt{E}(keV)$$

E being in keV. The part due to the Doppler broadening could be decomposed in three terms due respectively to the opening angle of the detector ($\Delta\theta$), the uncertainty concerning the direction of the source ($\Delta\theta_r$) and its absolute velocity value ($\Delta\beta$) .

1. $\Delta E_{(\Delta\theta)} = \int_{\theta-\Delta\theta}^{\theta+\Delta\theta} \frac{\partial E}{\partial\theta} d\theta = 2E\beta\sin(\Delta\theta)\sin(\theta)$
2. $\Delta E_{(\Delta\theta_r)} = \int_{\theta-\Delta\theta_r}^{\theta+\Delta\theta_r} \frac{\partial E}{\partial\theta} d\theta = 2E\beta\sin(\Delta\theta_r)\sin(\theta)$
3. $\Delta E_{(\Delta\beta)} = \int_{\beta-\Delta\beta}^{\beta+\Delta\beta} \frac{\partial E}{\partial\beta} d\beta = 2E\Delta\beta\cos(\theta)$

1. Effect of the opening angle of the detectors

In order to get an idea about the most suited opening angle for a detector unit, several curves have been drawn based on the first equation for the Doppler broadening. To get $\Delta E_{\Delta\theta}/E$ constant ($= k_{\Delta\theta}$) as a function of θ, $\sin(\Delta\theta)$ is given by:

$$\Delta E_{(\Delta\theta)}/E = k_{\Delta\theta} = 2\beta\sin(\Delta\theta)\sin(\theta) \Rightarrow \Delta\theta = \arcsin\left(\frac{k_{\Delta\theta}}{2\beta\sin\theta}\right)$$

Figure 3 shows the segmentations which ensure $\Delta E_{\Delta\theta} = 1$ and 3% for $\beta_{max} = 3\%$ and 10%.

Remarks:

- A 2 inches detector (2*25.4mm) at a distance of 15cm gives $\Delta\theta = 9.61^o$.
- To get $\Delta\theta = 3^o$ with a 2 inches detector, the distance from the target is 48.4cm.
- Taking into account an intrinsic resolution about 3% (LaBr3 at 662 keV), the total broadening (if the recoil velocity is perfectly known) is 3.16 and 4.24 keV respectively for $\Delta E_{\Delta\theta}$ equal to 1% and 3%.

[1] Talk of Joël Pouthas, PARIS meeting 14-15 May 2007

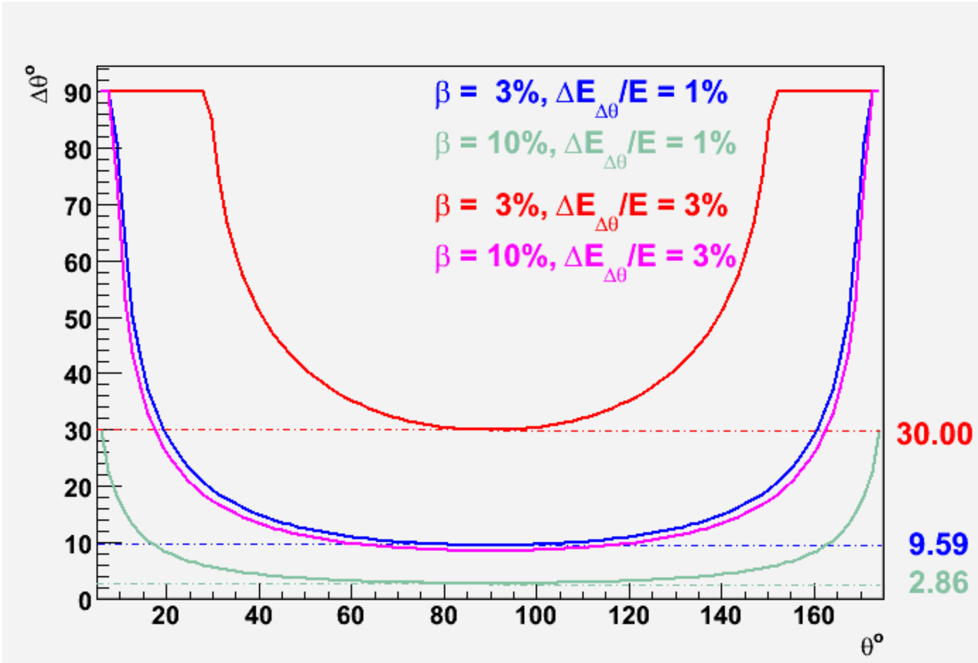

Figure 3: Required opening angle for the detectors as a function of θ for given β and given $\Delta E_{\Delta\theta}/E$.

2. Effects due to uncertainties concerning the recoil

The two contributions ($\Delta E_{(\Delta\theta_r)}$ and $\Delta E_{(\Delta\beta)}$) have been estimated for $\beta_{max} = 0.1$ and two values of $\Delta\theta_r$: 5^o and 10^o. The error on the velocity ($\Delta\beta$) has been set to 0.01 (see Fig4).

These parameters represent probably the worst situations regarding the PARIS physics cases. With a recoil velocity less than 5% and a reasonable value of $\Delta\theta_r < 5^o$, both contributions remain below 1%. Moreover, these effects could be considerably minimized in case the true recoil velocity is determined event by event.

Pile-up

Pile-up occurs when several photons arrive at the 'same time' in a single cell. It depends on the number of $\gamma-$rays emitted by the source (multiplicity of the cascade M_γ), the fraction of solid angle covered by the cell (Ω) and its absolute efficiency (ε_a). Thus, the probability to detect one $\gamma-$ray is given by $\varepsilon_a\Omega$ while the probability (P_γ) to detect **no other** $\gamma-$ray of the cascade in the same cell is given by:

$$P_\gamma = (1 - \varepsilon_a\Omega)^{M_\gamma - 1}$$

assuming the simple case for which the absolute efficiency ε_a does not strongly depend on the $\gamma-$ray energy or all the transitions of the cascade have the same energy. Pile-up is crucial for the first shell (in case Paris will be composed of several layers) that will be used as a spectrometer and/or to measure multiplicities. Thus, it concerns mainly energies in the range [0,2MeV]. For that range, a relatively high value of ε_a (0.9) is assumed for the forthcoming calculations. For the multiplicity of the cascade M_γ, several values are assumed from 10 to

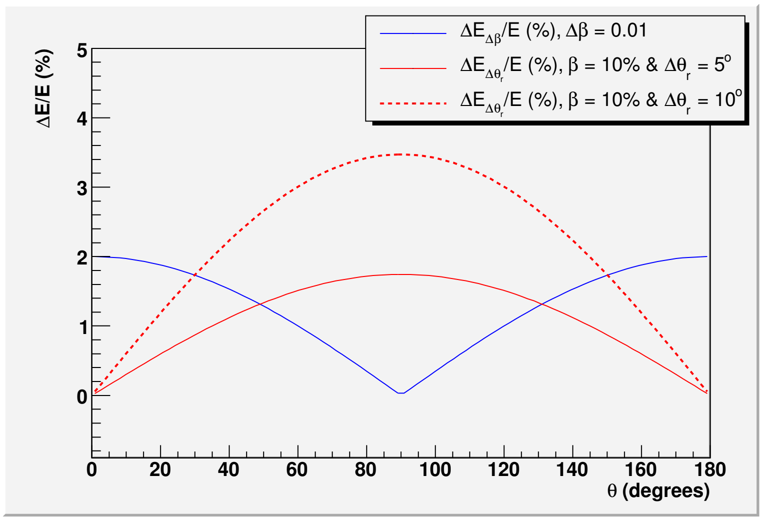

Figure 4: Doppler broadening due to the uncertainties concerning the recoil for several parameters

100. The maximum expected for Paris is probably around 40. However, to get an idea on how pile-up becomes important when increasing the counting rate, an upper limit of 100 has been fixed. For the fraction of solid angle (Ω) subtended by a single cell, it varies between two values determined by a cylinder with a diameter of 1 inch and 2 inches placed respectively at 20 and 10 centimeters from the target:

- $1''$@ 20 cm gives $\Omega \sim 1 \cdot 10^{-3}$
- $2''$@ 10 cm gives $\Omega \sim 1.5 \cdot 10^{-2}$

Remarks:

- $1''$@ 10 cm gives $\Omega \sim 3.4 \cdot 10^{-3}$
- $2''$@ 20 cm gives $\Omega \sim 3.9 \cdot 10^{-3}$

The results are displayed in Fig5 in which is represented (P_γ) for several multiplicities (from 10 to 100 by step of 10). For $2''$ cells placed at 20 cm from the target and a mean multiplicity of 40, the pile-up is slightly greater than 10%. Of course, these estimations suppose the time between two reactions is larger that the dead time needed to readout the fired cells. As well, a single $\gamma-$ray may fired several cells (Compton scattering, shower in case of GDR-like $\gamma-$ray). It is then a crude estimation of the pile-up (useful to get a reasonable idea of what should be the segmentation of the array) that has to be investigated more precisely with Geant4 simulations.

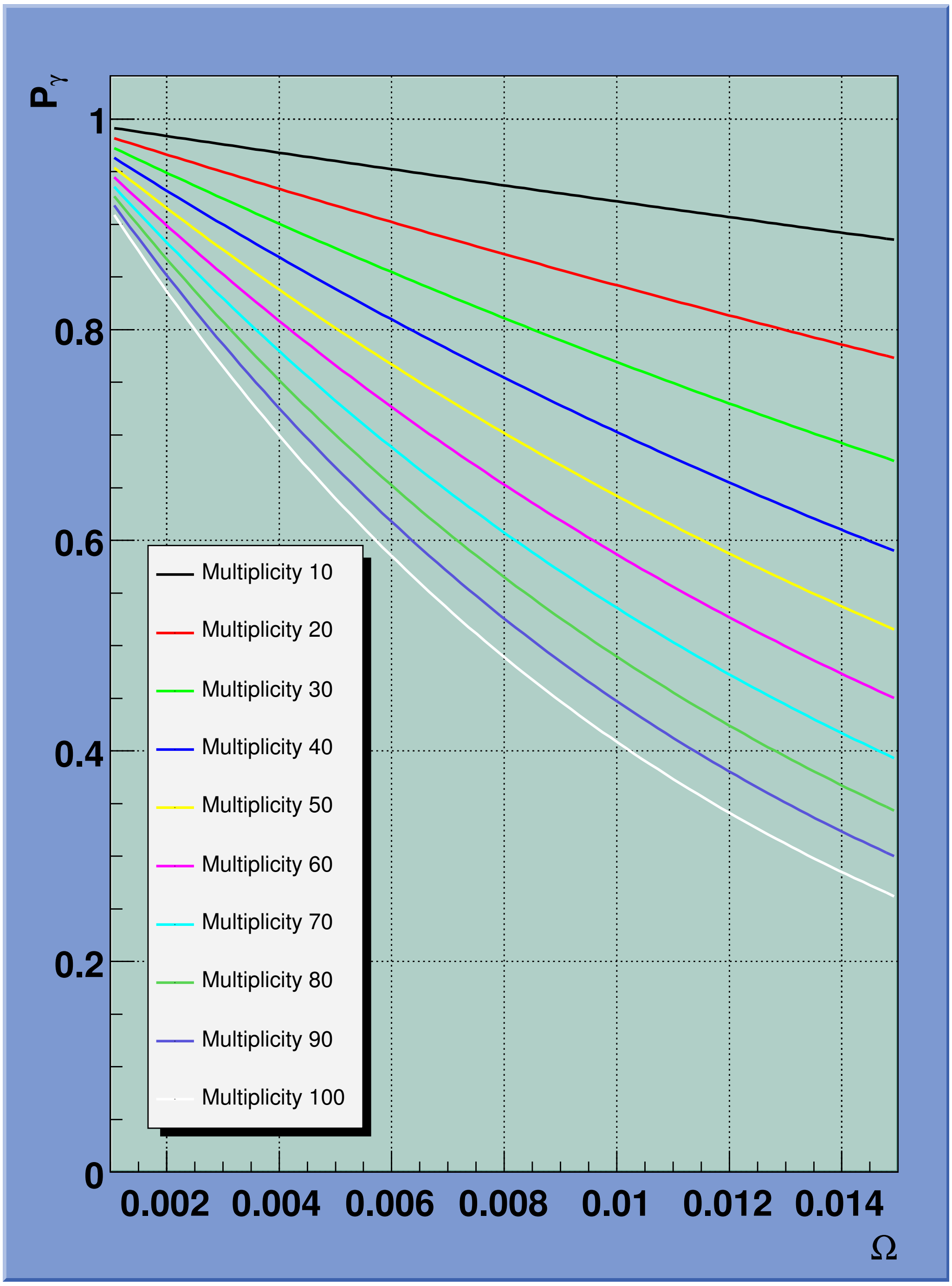

Figure 5: Probabilty P_γ to detect **one and only one** $\gamma-$ray from a cascade of M_γ $\gamma-$rays as a function of Ω for $\varepsilon_a = 0.9$.