Investigating Different Geometries for PARIS

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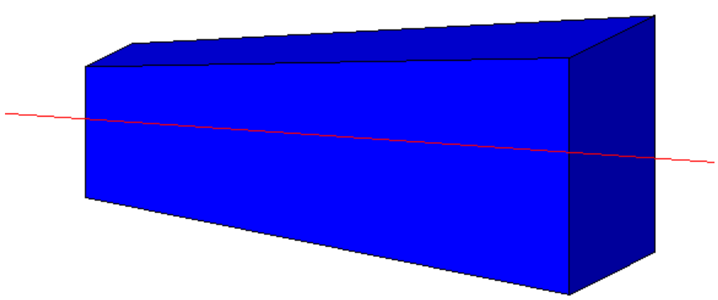
Abstract

This short report investigates the different geometries that are potentially available to be used in the PARIS calorimeter. The first set of simulations show the relative absorption efficiencies and discrepancies in energy resolution for the crystals, the parameters of which were fixed. Thus, their volumes would depend greatly upon which geometry was used. In the second half of the report, the same volumes were used for all of the tested geometries and the outcome from these results are presented and commented upon.

Absorption Efficiencies

Up until now, we have only investigated the idea of the detectors being in a cubic arrangement. However, typically, one would arrange detectors into a geodesic, 4π distribution that would yield a higher amount of absorption and lower the effects of dead space. With this in mind, preliminary tests were conducted on two shapes; a conical shape with a thickness from its frustum to its base, and a truncated pyramid. These shapes were originally tested on their own as individual segments and then compared to the rectangular shape recently proposed for the cubic configuration of the PARIS calorimeter. The face incident to the incoming gammas from the source were kept the same size, so that the projection of radiation onto the faces of each of the geometries was roughly equal. The source distance was kept at 15cm for all simulations.

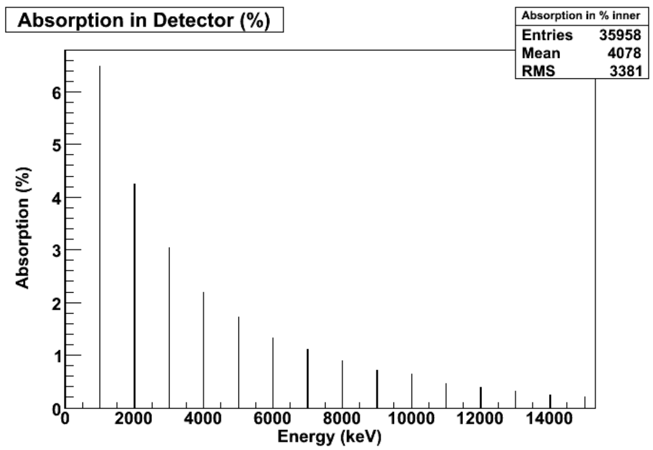


Truncated Pyramid

The first shape to experiment with was the Truncated Pyramid shape. Typically, This geometry is used in isotropic detectors, where a radial distribution can be approximately achieved. In reality, the $LaBr_3$ crystals are naturally hexagonal and very expensive to purchase and manufacture into this shape, but for the sake of demonstrating the efficiency of absorption, these were neglected.

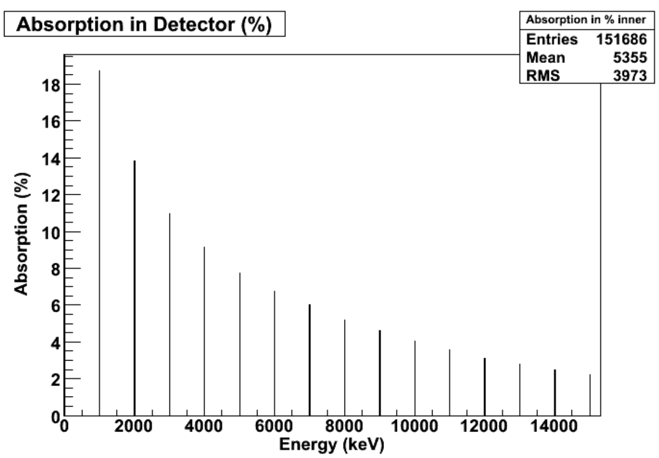
Now, we ask what results would we expect to get for a trapezoid that has the same dimensions on the incident face (1"x1") ,but has a length equivalent to that of the rectangle detector used in the cubic arrangement (4") give us? One would immediately assume that due to the geometry of the trapezoid the absorption would be better than the cube.

For a trapezoid with a length of 4” and a face of 1”x1” we have the graph below.



Looking at the graph, we see a low amount of absorption all around. At 1MeV, ~6.5% of the incident gammas are absorbed. This number then tails off rapidly, and at 15MeV, less than ~0.2% of gammas are absorbed, which is very poor. This is an interesting result as one would assume that by having the slopes of the truncated pyramid at an angle by which all the incident energy would hit the face, we would have a higher efficiency. However, this is not the case, and I think it is an interesting situation.

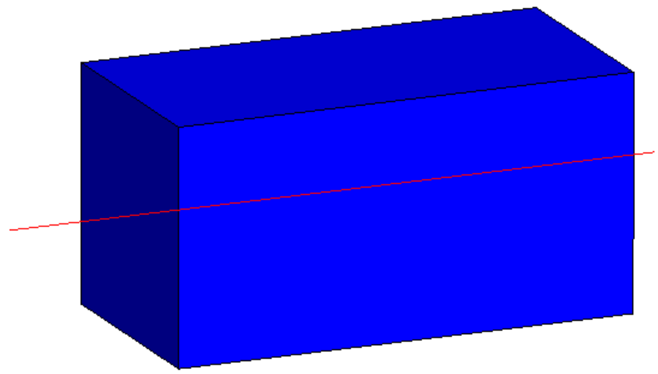
A similar simulation was run increasing the dimensions of the truncated pyramid by 2, so now its frustum is 2” and its end at 8.55, calibrated to include all the gammas in the projected beam.



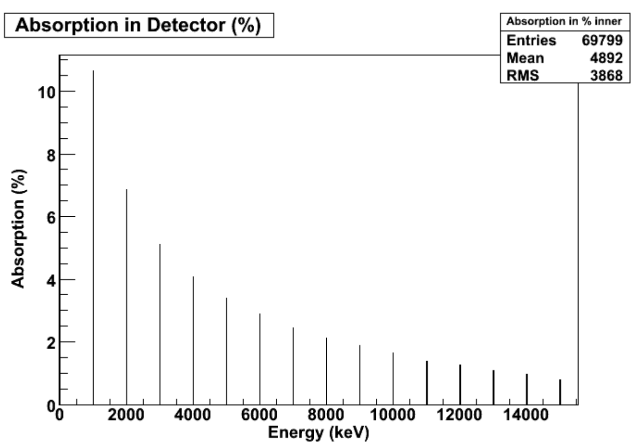
After running the same number of hits we see a higher amount of absorption. There is a peak of ~19% absorption for 1MeV gammas, compared to ~6.5% last time, a dramatic improvement. The absorption trend after drops off a lot slower than the previous simulation implying that more energies are being absorbed by the crystal. At 15MeV, we see around 3% of incident gammas interacting with the crystal, an absorption efficiency that is 15 times better at this energy for this volume than the previous run.

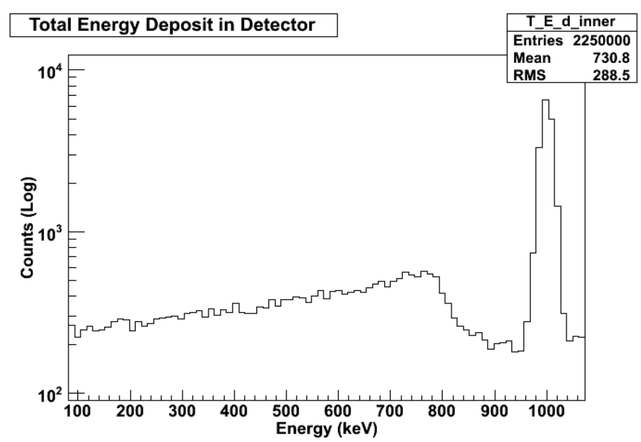
In summary, it would appear at first glance that this geometry might not be as efficient as the cube in terms of absorption efficiencies. We would assume that tiling this segment around a sphere would not only eliminate dead space, but increase the amount of absorption. However, as we will see, the amount of gammas interacting with similar sized detectors, of a less volume, is more efficient than this setup, which is a very interesting result. The next step would be to observe this comparison by simulating similar situations with the rectangular crystals.

Rectangular Geometry



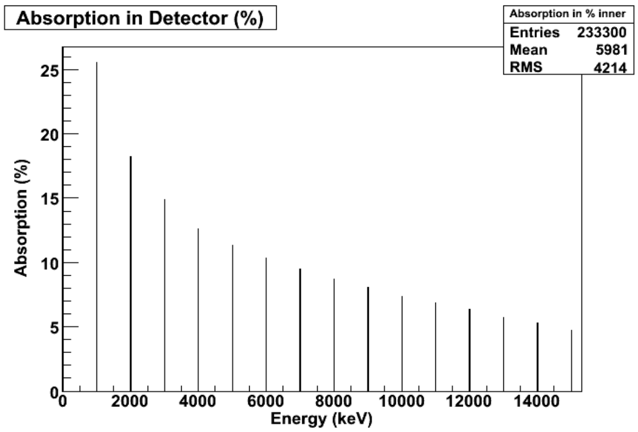
We look now to the original cubic/rectangular geometry. It was proposed that a more appropriate crystal to use in terms of absorption efficiency and cost at the PARIS meeting in May, is a $LaBr_3$ crystal, 1”x1”x4” in size. The simulations explored just one segment of this crystal at the said size, and then increased it to 2”x2”x4”. One would expect the second crystal size to bemore efficient as it has more volume, and thus a better chance at containing more gammas.

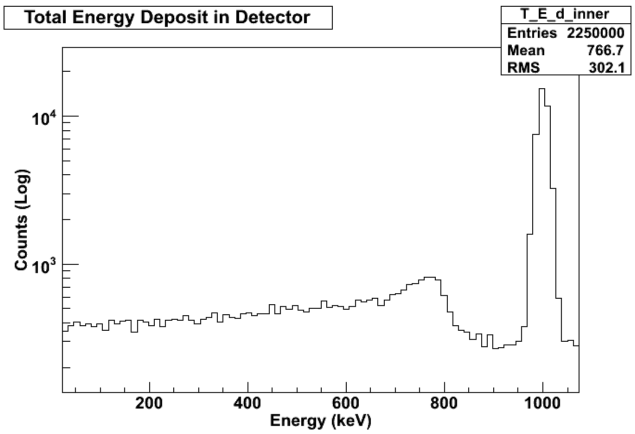




As one can see, after 150000 hits, we observe ~11% of incident low energy gammas (1MeV) are absorbed within the crystal. This absorption trend drops off sharply with increasing energy due to the increase in energy and also becauseof the detectors volume. At higher energies (e.g 15MeV) only ~1% of gammas are contained within the detector. The other graph shows a plot of counts (on a logarithmic scale) against Energy (at 1MeV). The energy resolution is set to roughly that of $LaBr_3$. We see one escape peak, this is due to energy smearing being incorporated into the simulations. I will briefly touch upon resolution in this case only, postponing the resolution of the crystals for the two different geometries until the end where this will be summarized and detailed comparisons drawn.

For the 2”x2”x4”, similar tests were conducted with the same energies. The results are as follows:

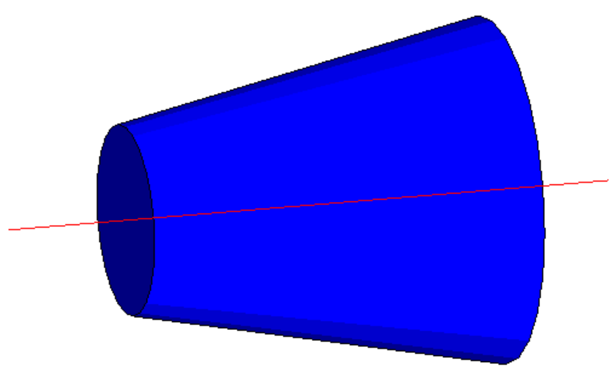




As one can see, the absorption is much higher than the previous simulation using this geometry. We now see a peak of ~25% at 1 MeV where a quarter of all gammas incident upon the crystal at this energy are absorbed. At 15 MeV, we see ~6-7% absorption, which again, is a vast improvement from the previous measurements. The spectrum is better too. The escape peak is more well defined instead of looking like “bump”, meaning better resolution.

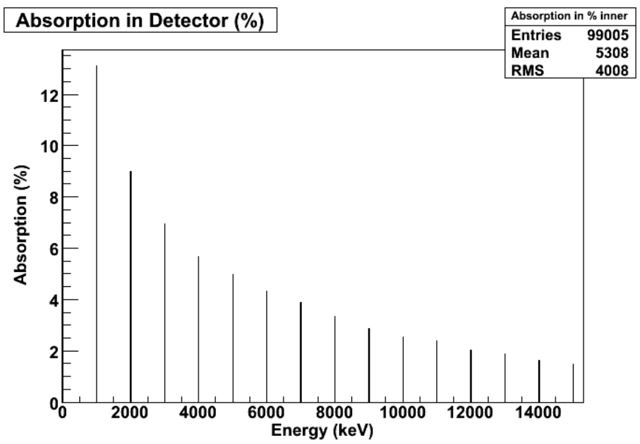
Cone Geometry

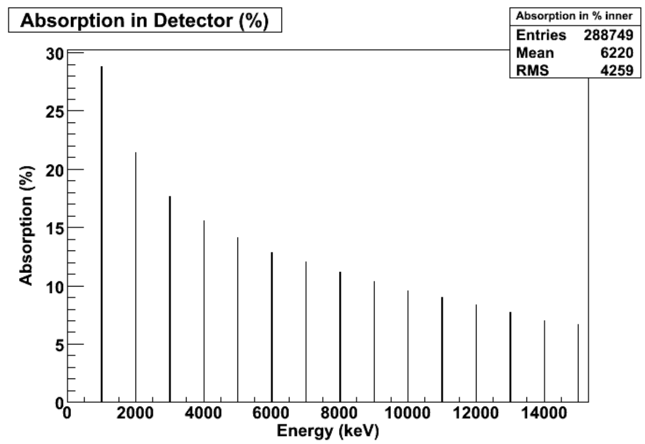
The final scenario would be to test the cone design, similar to the design below. It is a conical pyramid, with the top cut off revealing a frustum with a diameter of 2.54cm for the plane incident to the source. The bottom of the cone has a diameter of 4.62cm, which was calculated so that the projection of the beam followed the geometry of the cone, as was done previously with the other two designs. The length is 4” long to keep close to what was calculated with the trapezoid and cubic geometries.



I have used values that were used to calculate the square truncated pyramid, so I know immediately that the beam projection is larger than the area of the incident face, allowing for some gammas to hit the slopes of the detector as the beam projection is not completely trained upon the face of the geometry.

Now we look at the absorption efficiencies for both types of cone where the face incident to gammas was 1” and 2” in diameter respectively and had their length fixed at 4”.





Looking at the graphs we observe that for the smaller cone, 13% of 1MeV gammas are absorbed within the detector. This is the highest amount of gammas absorbed at this energy for any of the geometries that we have investigated so far. At higher energies, we see 2% of 15MeV gammas being absorbed. For the larger cone geometry, we see a very high amount of absorption. For 1MeV gammas, 29-30% of them are absorbed. This trend decreases at a shallower rate, meaning more absorption is taking place for all incident energies. At higher energies (<10MeV), we see absorption rates of between 7-14%, which is a much bigger improvement from anything we have seen previously.

Energy Resolution for the Geometries

An interesting thing to look at would be the effects of the energy resolution due to the geometry of the pyramid design. A value, "A" is used to generate the energy resolution of 3% as is typically seen for $LaBr_3$. This was found using an approximation found by Michal of the form

$$FWHM = A * E^{-1/2}$$

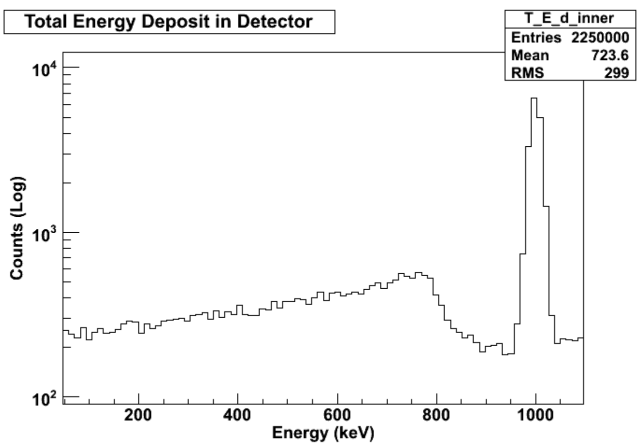
This is a better approximation for high energy gamma rays but was used as an approximation for the generated resolution graphs. The same value of A = 0.12 was kept for all energy resolution graphs at 1MeV to be consistent. The data, with errors are given in the following table:

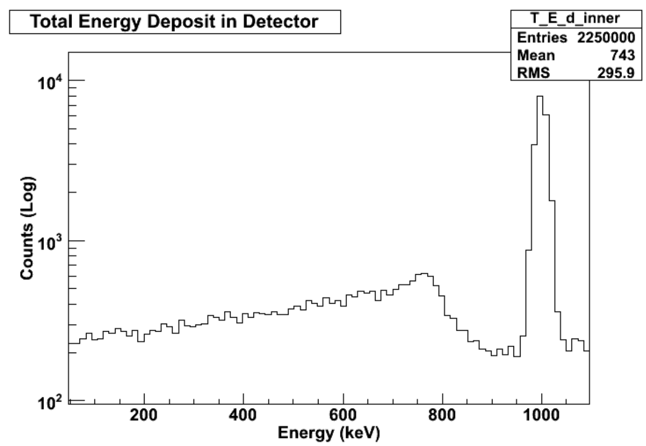
Solid Dimensions	σ (ROOT)	Error +/- σ	Resolution (%)	Error +/- FWHM (%)
1"x1"x4" Cube	12.56	.083	2.95	0.02
2"x2"x4" Cube	12.31	.052	2.89	0.01
1"x1"x4" Trap.	13.04	.122	3.06	0.03
2"x2"x4" Trap.	12.51	.060	2.94	0.01
1"x1"x4" Cone	12.49	.073	2.94	0.02
2"x2"x4" Cone	12.14	.047	2.85	0.01

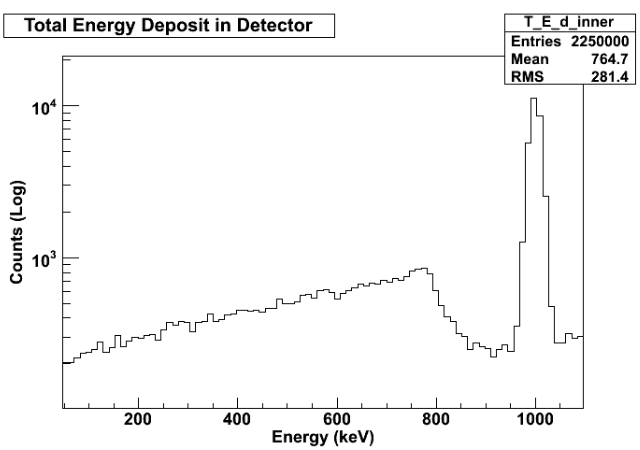
As one can see, these values do not fall on 3% within error, which means that the parameter needed to be adjusted for each measurement. However, all the resolutions seem to be within .15% which is close enough to the values we needed. The value of sigma was calculated by fitting a Gaussian curve in ROOT to the photo-peak of one of the incident energies (in the table above, 1MeV). This was then compared to calculations for 3% resolution

The error in σ is given from the fits in ROOT. The errors and resolutions were then calculated consequently. Most of the fitted values for sigma and the resolution fall within 2% of the 3% value which is a very good approximation. The resolution was not calculated for 15MeV, but can be done so in a similar fashion to what has been done above by changing the value of "A". As the value of A remained the same, the resolution of the graphs for 15MeV gammas show a resolution of around 0.69 ± 0.018%.

These values are now used in the two sets of energy resolution graphs for both sizes of geometries. First, we will examine the smaller 1"x1"x4" configuration.

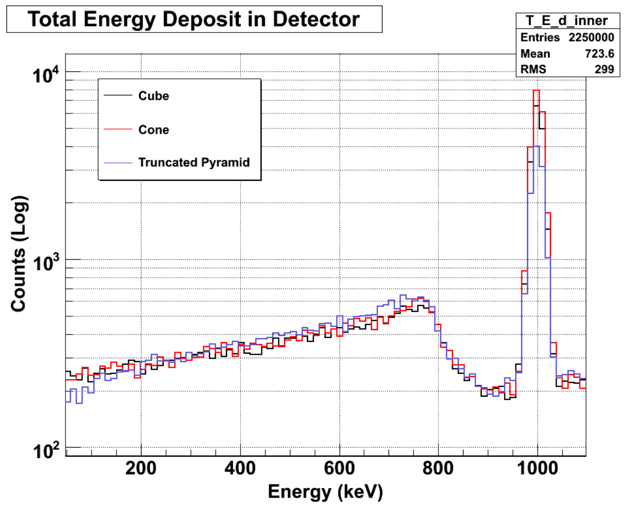


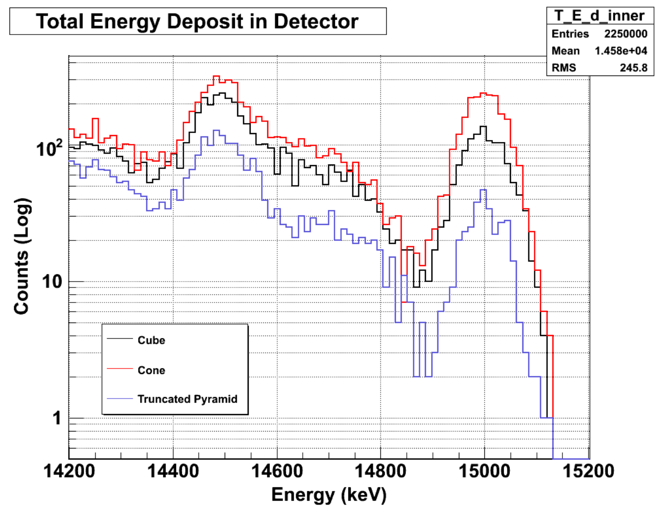




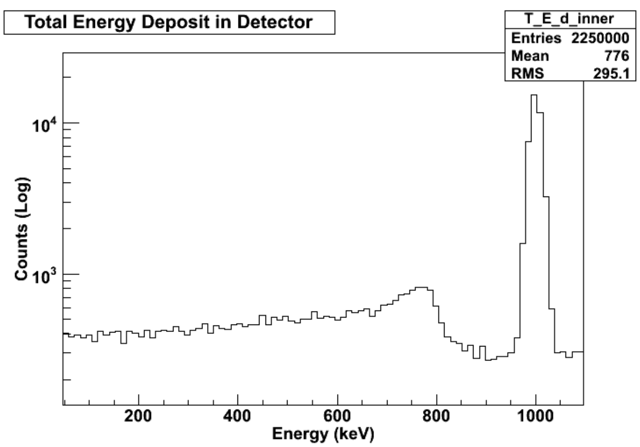
The top left graph is the Energy resolution of a 1"x1"x4" cube. The graph on the top right (above) is the energy resolution spectrum for a 1MeV gamma incident on a cone with inner diameter of 1". The last spectrum below (left) is a spectrum for the same incident energy but onto a truncated pyramid shape whose frustum measures 1"x1" in area. All the lengths of these solids were kept fixed at 4".

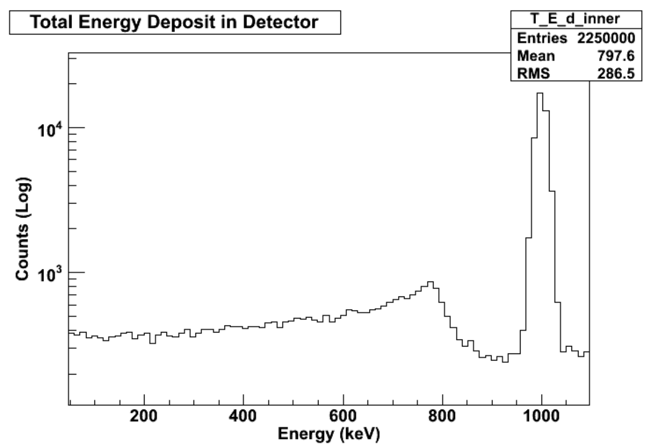
All the graphs have the same binning to be consistent. Looking at the graphs one notices slight discrepancies in the nature of the spectrum and the clarity of the escape peak. Due to energy smearing, we can only properly view 1 escape peak. The discrepancies are outlined in a graph overleaf, which superimposes all there graphs in order to help identify which is better. Contrary to what might be expected, the truncated pyramidal design yielded the worse resolution of the photo-peak, while the cone gave the best. One can particularly notice this difference with the first escape; clearly defined with the conical design, but not so much with the trapezoidal solid at 1MeV. The amount of counts and thus the binning, is poorer for the 15MeV graph where we can clearly see the differences in the geometries. At higher energies the trapezoidal shape is extremelypoor compared to the cone and cubic geometries.

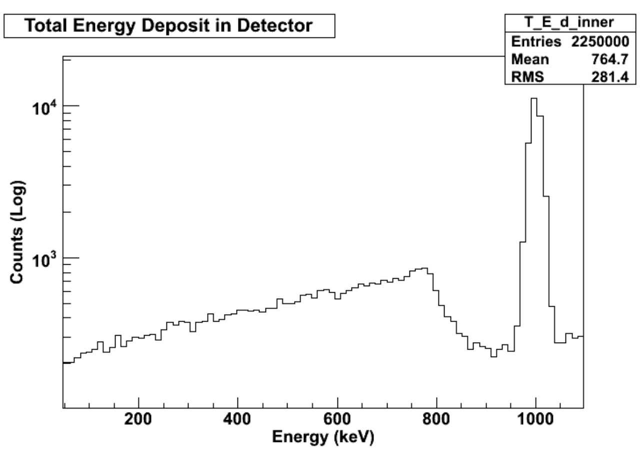




Now we investigate, similarly, the results of the larger 2”x2”x4” dimensions for the geometries.

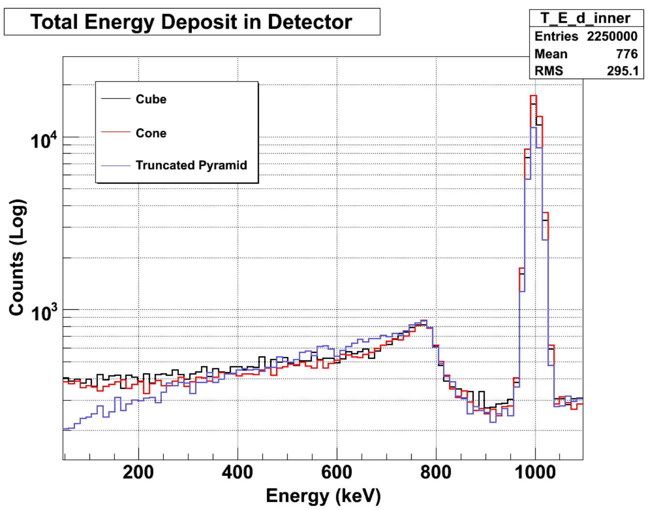


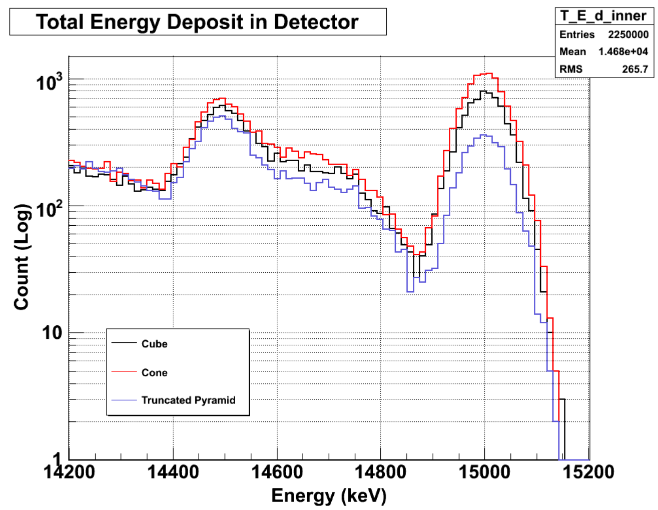




The top left graph is the Energy resolution of a 2”x2”x4” cube. The graph on the top right (above) is the energy resolution spectrum for a 1MeV gamma incident on a cone with inner diameter of 2”. The last spectrum below (left) is a spectrum for the same incident energy but onto a truncated pyramid shape whose frustum measures 2”x2” in area. All the lengths of these solids were kept fixed at 4”.

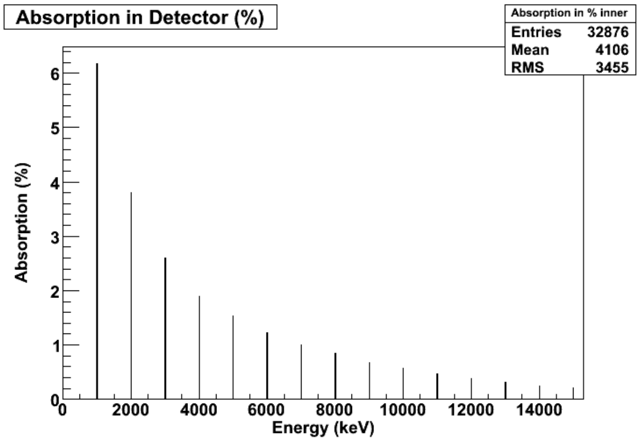
Looking at the graphs, one notices an increase in the resolution of the escape peak. It starts to take more of a peak shape instead of the bump seen previously. This is outlined clearly in a graph similar to what we saw on the previous page. The corresponding resolution graphs for an incident 1MeV gamma are superimposed to determine which shape gave the best resolution, with the parameters defined. The trapezoidal pyramid again gave the worst results, whilst the cone and cube define both the photo-peak and 1st escape peak well, with a bit more resolution being achieved with the cone design at 1MeV. For the higher energy graph at 15MeV, we see the discrepancies emphasized.The differences in the the 1st escape peak are rather small, compared to the photo-peak which shows, again, the stark difference between the trapezoidal geometry and the cone and cubic designs, which come out on top. Due to more counts from the increased volume size of the crystal, we begin to distinguish a 2nd peak, although still of a very poor resolution.

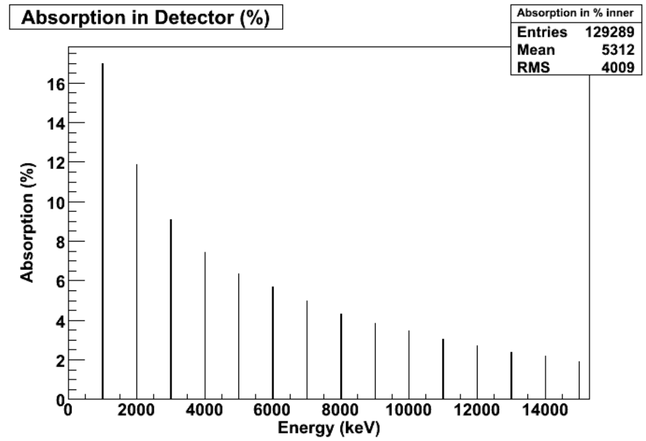


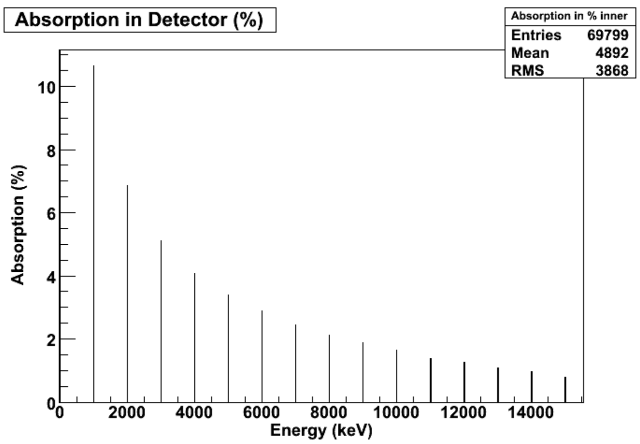


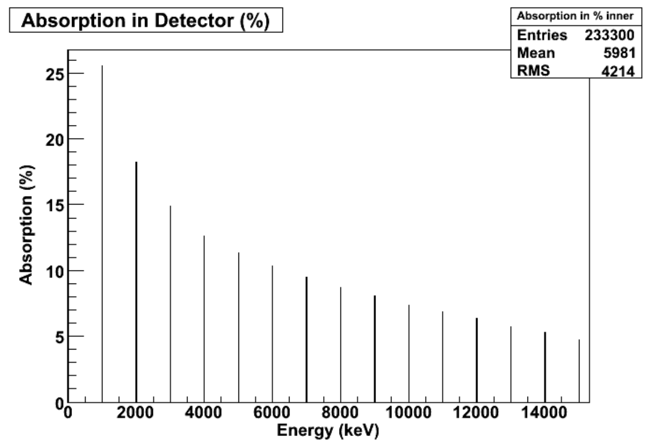
So choosing dimensions and thickness for all geometries has shown that the cone is the best geometry to use, followed closely by the cubic/rectangular design. It is hard to manufacture a cone out of $LaBr_3$, so a cost efficient and simple result would be to stick with the cubic geometry. The trapezoidal solid yielded a poorer amount of absorption and is thus not a solid that would be beneficial to use in the PARIS calorimeter, despite limiting the dead space by tiling well into a 4π, geodesic structure.

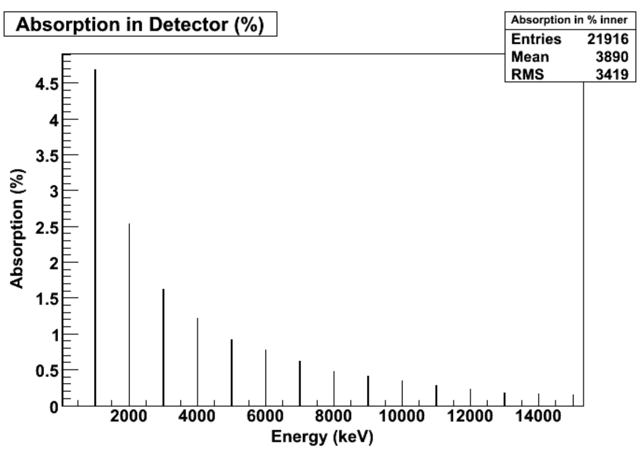
Now we look to experiment with the volumes equal for each of the geometries. This would mean that some lengths will be shorter than others. For a 1"x1"x4" cube the volume is 65.55cm^3, and for a 2"x2"x4" rectangular volume used for the cubic measurements, the volume is 262.19cm^3. Each of the geometries have been calibrated so that the projecion of the beam upon the setup geometry yields the maximum amount of counts.

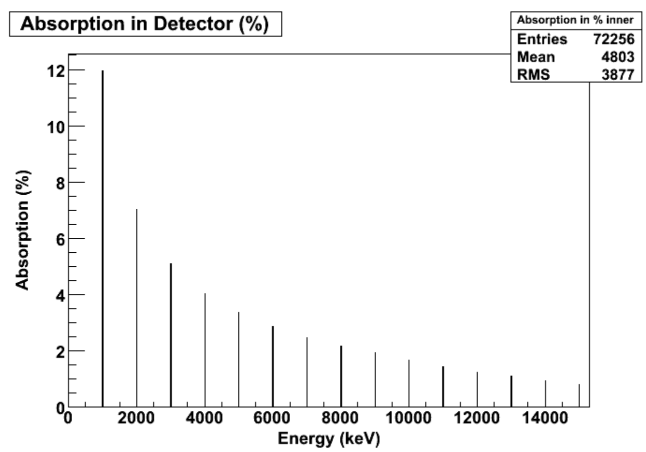










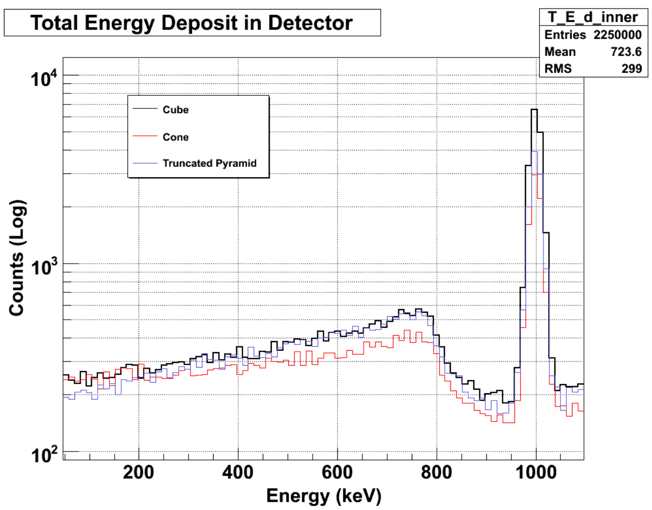


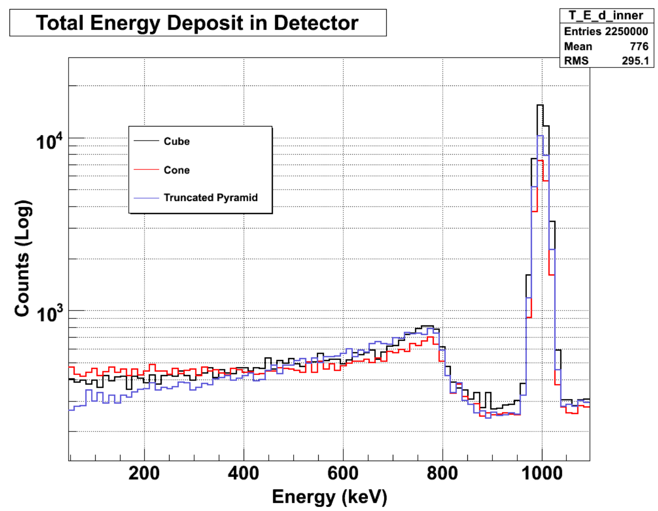
The graphs are laid out so that the first column are the smaller 1”x1”x4” measurements and the second column are 2”x2”x4” measurements, both easy to distinguish due to the increase in absorption efficiencies that one would expect foran extended volume size. The first row are the trapezoid/truncated pyramidal absorption graphs, followed by the cubic graphs in the second row and the cone measurements in the bottom row.

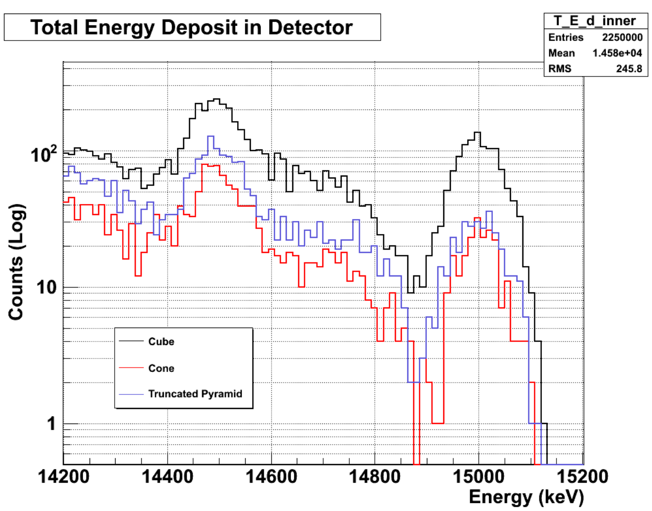
Energy Resolution

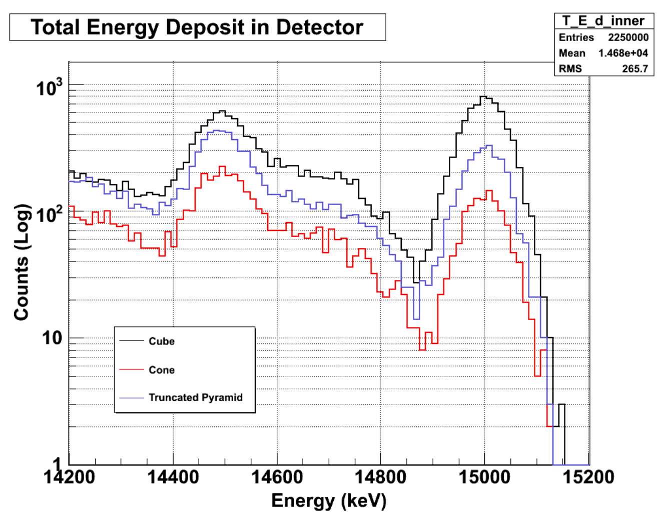
Now, we turn our attention to the efficiency of the detectors. Previously, I had all the energy resolution graphs shown and then summarized with a histogram of the stacked graphs to show discrepancies in resolution at 1MeV and 15MeV. However, I will only show the latter superimposed graphs at both 1 and 15MeV, as it can be assumed that a smaller volume will be tested compared to what we saw before therefore yielding similar traits in the energy resolution, although with a lower number of counts.

The first two graphs show the energy resolution for our three geometries superimposed for a 1MeV gamma. The best resolution goes to the cone design again which shows a clear, well defined 1^{st} escape peak, despite having a poorer count rate. This is likely to be due to the fact the the volume is small, and thinner than the previous simulations. For the larger 2”x2”x4” volume, the cone seems like a worthy competitor, especially at lower energies. It is, however, not as efficient in absorbing gamma as the cubic and trapezoidal shapes which both possess a higher count rate, despite having poorer resolution than the cone.









For the higher energy gammas (in our case 15MeV), we see that the cone produces good resolution in comparison to the other geometries, but a poor count rate. This is due to the very small face that was incident on the source when using the smaller 1”x1”x4” volume (graph on the bottom left). The high count rate seals the cubic design as the best volume to use due to its 2” advantage in thickness over the other geometries, despite having a poorer resolution. The graph to its right shows the absorption efficiency for the 15MeV gamma for the larger 262.19cm^3 volume used in the simulations. The 1st escape peak is well defined, however, it is clearly obvious that in terms of more counts the cubic design is best due to this extension of thickness. The cone however looks like it possesses the most resolved spectrum.

Looking at the graphs, I have again made a table similar to the first half of the report where I have calculated the resolution that each volume and geometry generate. No calculations for resolution were calculated for the 15MeV gammas, but can easily be done and would be a short assignment to complete the report if one wanted to expand furtherto include more detail. For now though the chart will just contain the resolution for the fixed volumes when absorbing 1MeV gammas.The value for “A” would need to be changed if wanting to obtain 3% resolution of the 15 MeV graphs as their resolution is currently ~0.69%.

Solid Dimensions (Volume)	σ (ROOT)	Error +/- σ	Resolution (%)	Error +/- FWHM (%)
1”x1”x4” Cube	12.56	.083	2.95	0.02
2”x2”x4” Cube	12.31	.052	2.89	0.01
1”x1”x4” Trap.	12.99	.120	3.05	0.03
2”x2”x4” Trap.	12.59	.065	2.96	0.02
1”x1”x4” Cone	12.70	.139	2.98	0.03
2”x2”x4” Cone	12.72	.070	2.99	0.02

Again, the resolution is very good, the value of “A” is a good approximation when performing the energy resolution graphs as the values seem to stay within .10% of the resolution, despite the errors from sigma being much smaller than this. However, it is still a good approximation.

Summary

Looking back on the simulations performed in this analysis on the different potential geometries to have for the PARIS calorimeter. It is safe to say that the cone geometry is the best in terms of resolution. The cubic design has a resolution less of that of the cone, but produces a higher count rate, especially in the second half of the simulations where the volume was kept fixed for all shapes, which yielded smaller volumes for the other geometries. The report can be extended to analyze more realistic situations with the geometries tested, so as to incorporate segmentation to see what role that would play. Although one would assume similar trends in both resolution and absorption efficiency for these simulations, the lower count rate would depend on what gapsizes would be incorporated. Another thing one can do to expand on this report, would be to look at the where the shifted-cube design would fit within the results observed.

It should be noted that although the cone geometry gave the best resolution, its not cost effective as the cost and amount of work to produce a crystal like this would be very difficult and expensive due to the inner chemical structure of $LaBr_3$. However, more work on the cubic geometry would allow us to better understand if this geometry is the best to be incorporated in the arrangement of the final designs of the PARIS calorimeter.