# Jacobi and Poincaré Shape Transitions at High Temperatures and High Spins

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- **At high temperatures, the total nuclear energy can be approximated by the macroscopic energy expression only**

- **The angular momentum effects can be treated, to the first approximation classically**

$$E_{total}(\{def.\}; I) = E_{macro}(\{def.\}) + \frac{\hbar^2}{2\mathcal{J}\{def.\}} \cdot I(I+1)$$

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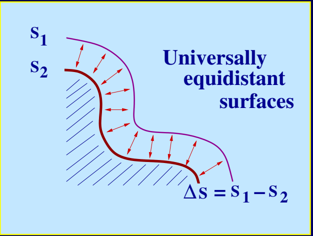
Using the macroscopic energy as optimal as possible will be of importance: in our case ⟶ the LSD Model

# Heuresis of LSD: Universally Equidistant Surfaces

- Let a closed surface be given together with an infinity of surfaces obtained, by construction, through a point-after-point perpendicular shift to the distance $\Delta s$ ('Steiner sheets').




- Let $\kappa \equiv 1/R_1 + 1/R_2$ be a local average curvature. Then the volume $\mathcal{V}$, the surface area $\mathcal{S}$ and the average curvature $\mathcal{L}$ can be expressed as:

$$\mathcal{V} \equiv \frac{1}{3}\int_\Sigma dS \vec{\nabla} \cdot \vec{r}; \quad \mathcal{S} \equiv \int_\Sigma dS; \quad \mathcal{L} \equiv \int_\Sigma dS \left(\frac{1}{R_1} + \frac{1}{R_2}\right),$$

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satisfy, as one can demonstrate:

$$\mathcal{L}(s) = \frac{d\mathcal{S}}{ds} = \frac{d^2\mathcal{V}}{ds^2}$$

and consequently the following Taylor series expansions hold

$$\mathcal{V}(s) = \mathcal{V}_0 + \mathcal{S}_0\, s + \tfrac{1}{2}\mathcal{L}_0\, s^2 + \ldots \qquad \mathcal{S}(s) = \mathcal{S}_0 + \mathcal{L}_0\, s + \ldots,$$

- The nuclear surface energy comes from the nuclear matter contained in a certain surface region whose magnitude is determined by its diffusivity.

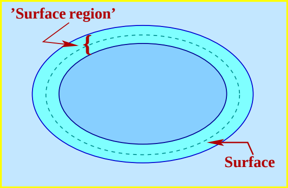


Figure: *For Steiner sheets and relatively thin skin (small surface region) the amount of nuclear matter contained in the surface region is approximately proportional to the volume of the surface region.*

- The volume of the 'surface region', $\mathcal{V}_S$, is approximated by

$$\mathcal{E}_{surf} \sim \mathcal{V}_S \sim \int_{s_1}^{s_2} \mathcal{S}(s)ds \sim \int_{s_1}^{s_2} [\mathcal{S}_0 + \mathcal{L}_0 s]\, ds$$

# Conclusions for the Macroscopic Models (II)

- Repeat: The volume of the 'surface region', $\mathcal{V}_S$, is approximated by

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wherefrom

$$\mathcal{E}_{surf} \sim \mathcal{S}_0 \underbrace{(s_2 - s_1)}_{\mathcal{C}_S(Z,N)} + \mathcal{L}_0 \underbrace{(s_2^2 - s_1^2)}_{\mathcal{C}_L(Z,N)}$$

so that

$$\mathcal{E}_{surf} = \mathcal{C}_S(Z,N) \cdot \mathcal{S}_0 + \mathcal{C}_L(Z,N) \cdot \mathcal{L}_0$$

- Conclusion: The surface energy is split now into two terms, one proportinal to the surface area and one proportional to the average curvature

# Conclusions for the Structure of $\mathcal{E}_{surf}$

- The nuclear surface energy can be decomposed into at least two terms whose **A**-dependences are different: $\mathbf{A^{2/3}}$ and $\mathbf{A^{1/3}}$
- At *the same surface area* $\mathcal{S}_0$ two nuclei differing by average curvatures $\mathcal{L}_0$ and $\mathcal{L}_0'$, *will have different surface energies*
- Since the proportionality coefficients $\mathcal{C}_S(Z, N)$ and $\mathcal{C}_L(Z, N)$ are in fact *'functions of the nucleus'*, it follows that in two different nuclear regions the relative proportions of the surface-area term to the surface-curvature term will be in general different (e.g. vanishing surface-curvature)
- The surface energy is proportional to the *volume of the surface region*
- There is no *a priori* statement about the sign of curvature contributions

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# The Physics of the Nuclear Surface

- The fit of parameters of the extended formula to 2772 masses improves the results for the barriers by better than a factor of 4 (!!)

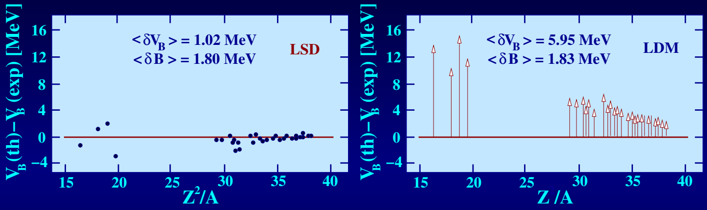


Figure: *Results of the fitting of the parameters to the experimental masses give simultaneously and improvement in the description of the experimental fission barriers (left); fit performed under the same conditions but without curvature terms ('traditional') is given for comparison on the right.*

About the Method Used in this Work:

# Macroscopic Energy Calculations

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- In the past, often the Yukawa-folded approach has been used;
- In such an approach the surface energy is obtained through a procedure using the Yukawa-folding function $F(|\vec{r} - \vec{r}'|, a)$
- The diffuseness parameter a serves to collect the contributions from the nuclear surface region only
- The folding procedure results in a dangerous loss of sensitivity with respect to high-order multipoles
- Also the fission barrier-heights especially for the lighter nuclei do not correspond well with the experimental data

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# Macroscopic Energy Calculations: Stiffness Pathology

- The folding procedure and the optimally fitted parameters both result in a characteristic loss of sensitivity with respect to high-order multipoles: Stiffness remains weak at increasing multipolarity

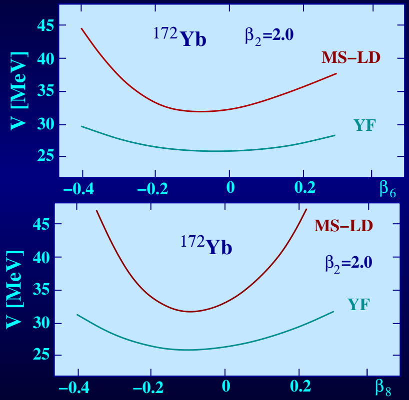


Figure: At large elongation, Yukawa-folded macroscopic energies depend relatively weakly on the higher order multipoles: $\beta_6$, $\beta_8$, $\beta_{10}$, etc.

# The Final LSD Macroscopic Energy Expression

- *Mass-fits are improved slightly with respect to other models but the fission barriers are improved considerably;*
- *The fission barriers involve large deformations where the curvature of the nuclear surface plays an important role;*
- *This significant improvement confirms the right physics:*

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\begin{aligned}
E_{lsd}(Z,N;def) &= b_{vol}\{1-\kappa_{vol}[(N-Z)/A]^2\}\,A \\
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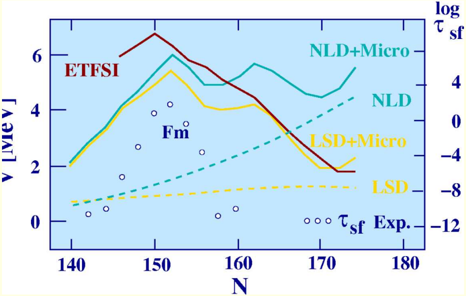
# LSD - Some Illustrations




Comparison of the model results: Extended Thomas Fermi with Skyrme Interaction (ETFSI), Lublin-Strasbourg Drop (LSD) and the 'traditional' one (NLD). The logarithms of the spontaneus fission half lives are given for qualitative comparison (right scale)

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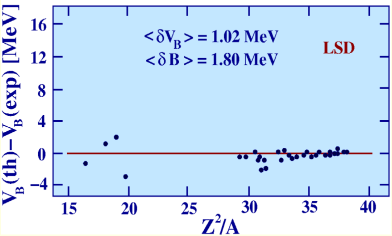
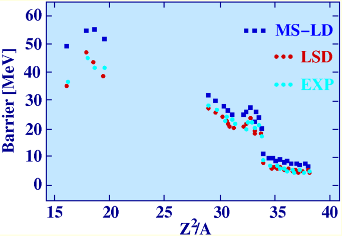
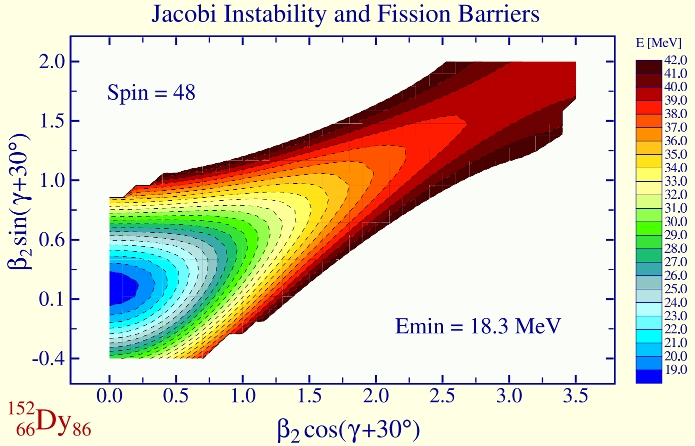



*Difference between theoretical and experimental fission barriers heights.*

*J.Dudek, K.Pomorski, Phys. Rev. C67 (2003) 044316*

# Jacobi Shape Transition in Dy




*$^{152}Dy$ -hyperdeformed nucleus.*

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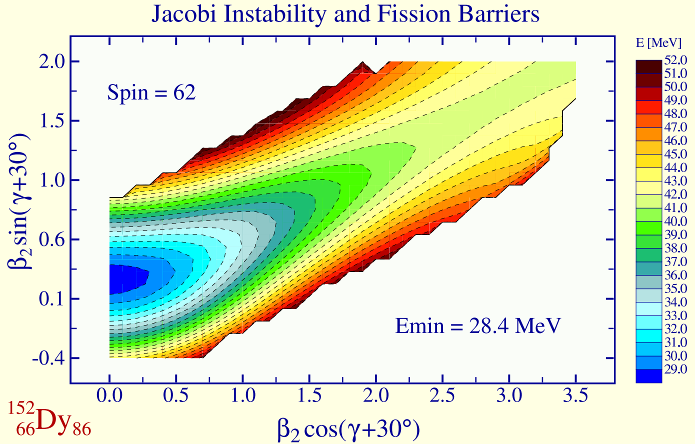


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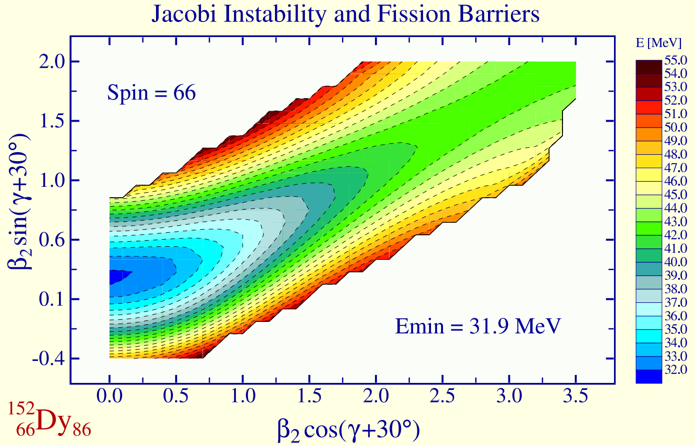


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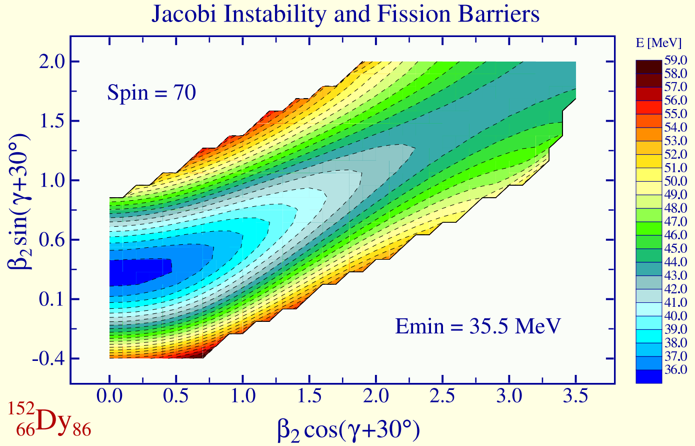


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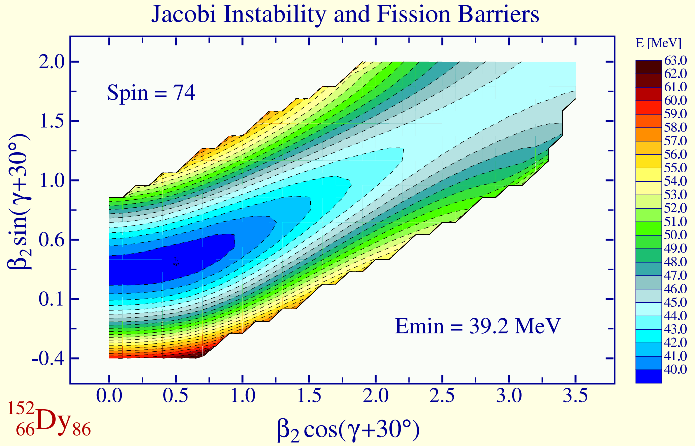


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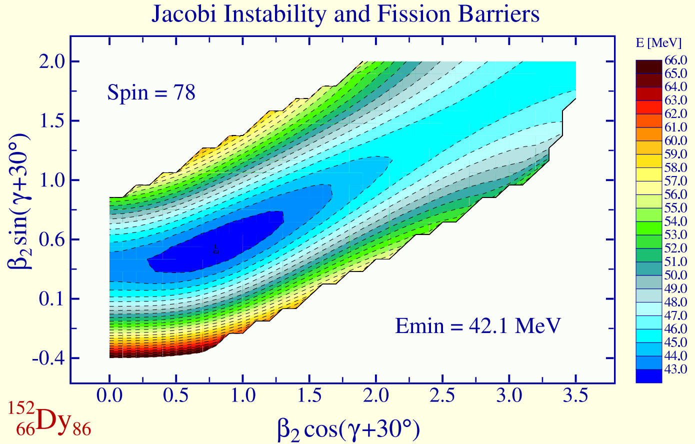


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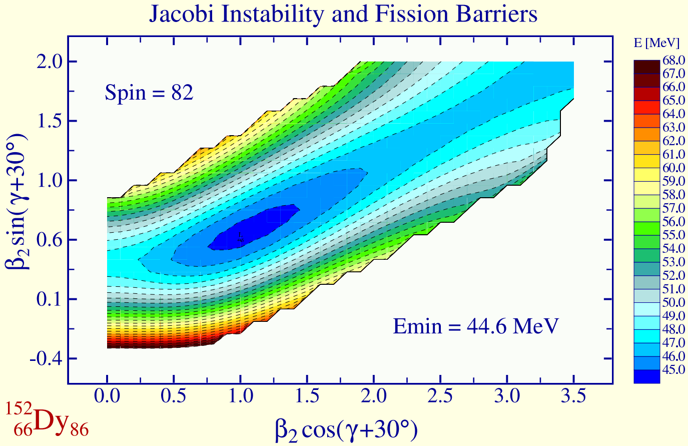


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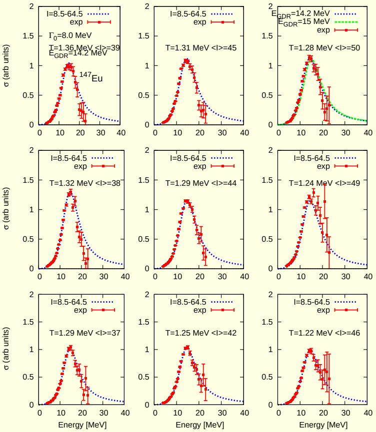
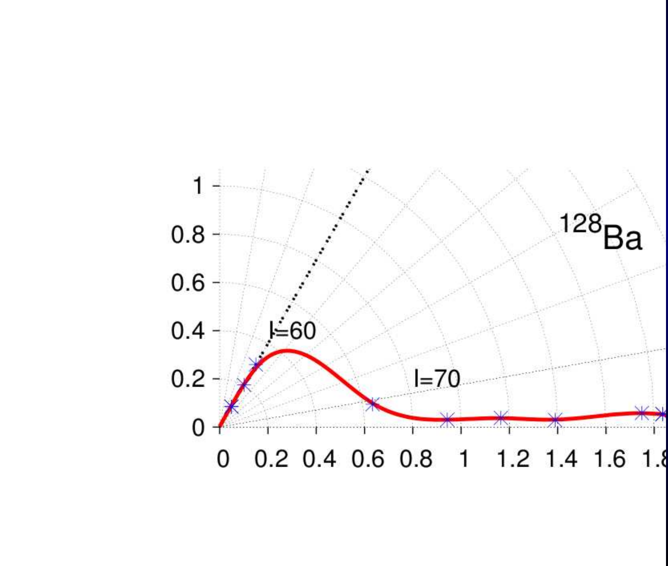
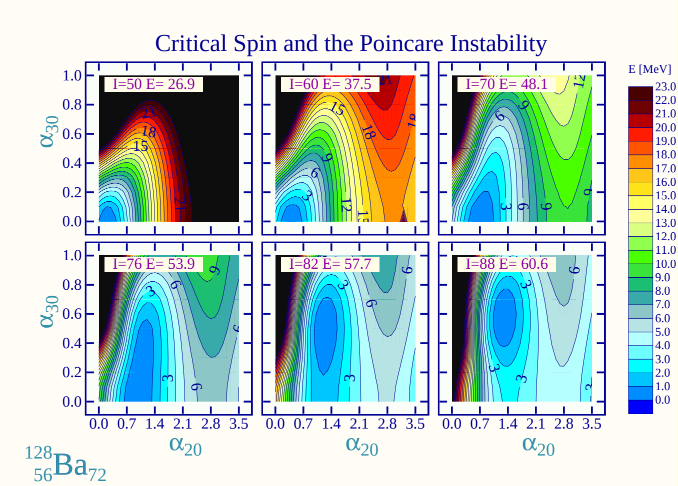
# LSD and Simulation of the GDR Profiles



# Jacobi Shape Transition - Spin Trajectory



# Poincaré Shape Transition




*Comparison of the evolution of axial and triaxial energy equlibrium for spins* $I = 50, 60, 70, 76, 82, 88$.

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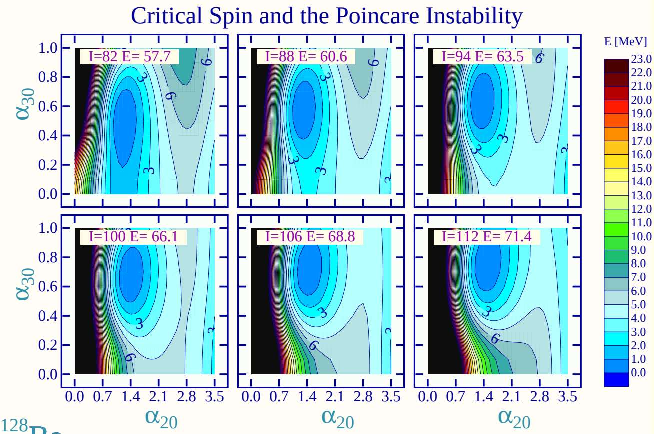
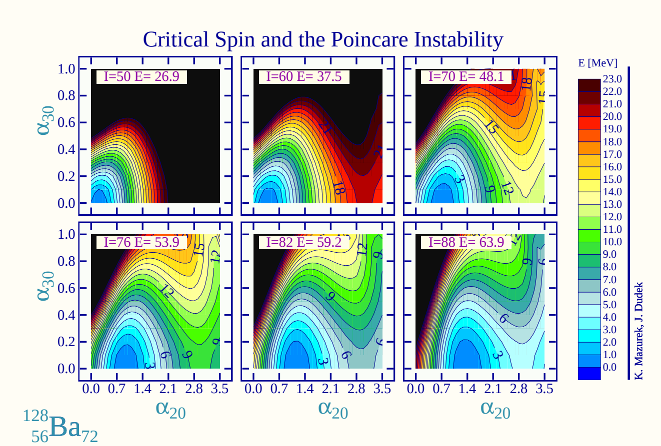


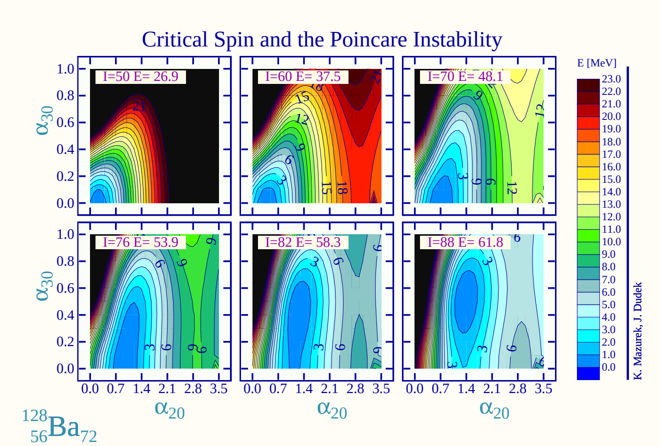

$^{128}_{56}Ba_{72}$

*Comparison of the evolution of axial and triaxial energy equlibrium for spins* $I = 82, 88, 94, 100, 106, 112$.

# Poincaré Transition and High Multipoles




*Stabilizing the barrier by minimization over additional deformation parameters:* $\alpha_{40}$.

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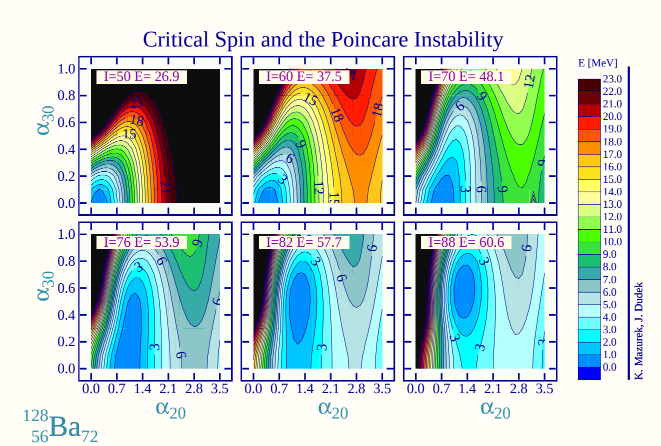


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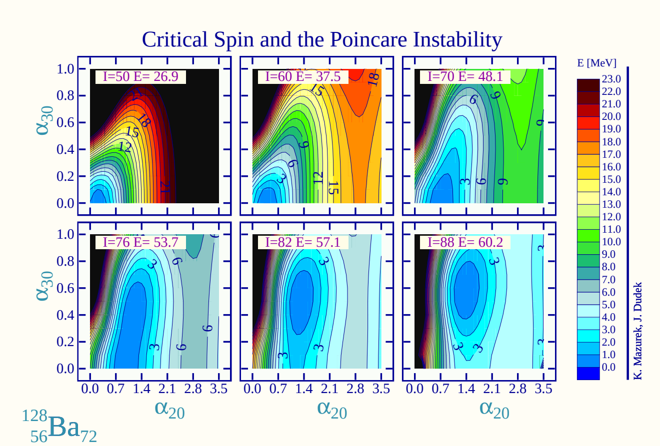


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## Summary and Conclusion

• The LSD model used here describes the fission barriers significantly better than the preceding models

• Consequently, it is well suited for the realistic calculations of the nuclear fission barriers at high temperatures

• It is also well suited for simulation of the spin-dependence of the classical energies as functions of deformation

• According to calculations the Poincaré transitions compete with the Jacobi shape transitions

• Since the Poincaré shape transitions arrive relatively sharply in function of spin - its experimental manifestation should be an abrupt increase of the fission-fragment mass-asymmetry with increasing spin

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This is a very well defined experimental program !

Collaborators: