

# **GEANT4 simulations of a single LaBr<sub>3</sub>(Ce) detector and large NaI(Tl) detector arrays**

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# Collaborators



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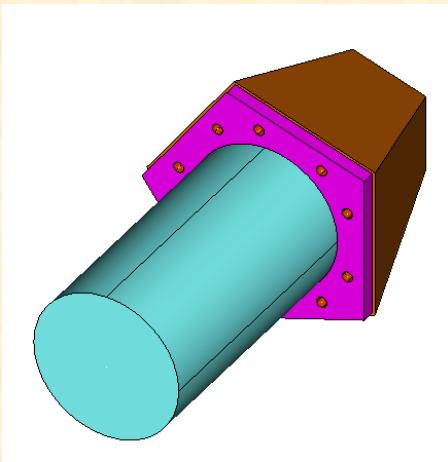
# Plan of the Talk

- GEANT4 simulations of large arrays of NaI(Tl) detectors in soccer-ball and castle geometries
- A comparative study efficiencies of LaBr<sub>3</sub>,NaI(Tl), BaF<sub>2</sub>.
- Close-geometry efficiency calibration and true coincidence summing correction



## Aim

- To calculate the detection efficiencies of the individual detectors and the entire  $4\pi$  array using GEANT4 and comparison with measurements.
- To carry out efficiency measurements and GEANT4 simulations for a smaller array of 14 straight NaI detectors of hexagonal cross sections packed in castle geometry and the comparison of the results with the  $4\pi$  array.
- To calculate fold distributions for different gamma multiplicities for both the 14 elements and the  $4\pi$  array.



Pentaagon



Hexagon



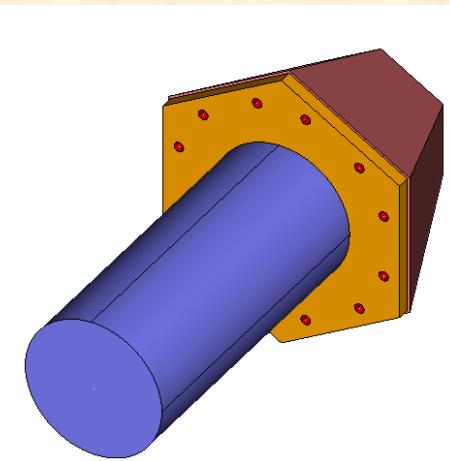
## Specifications

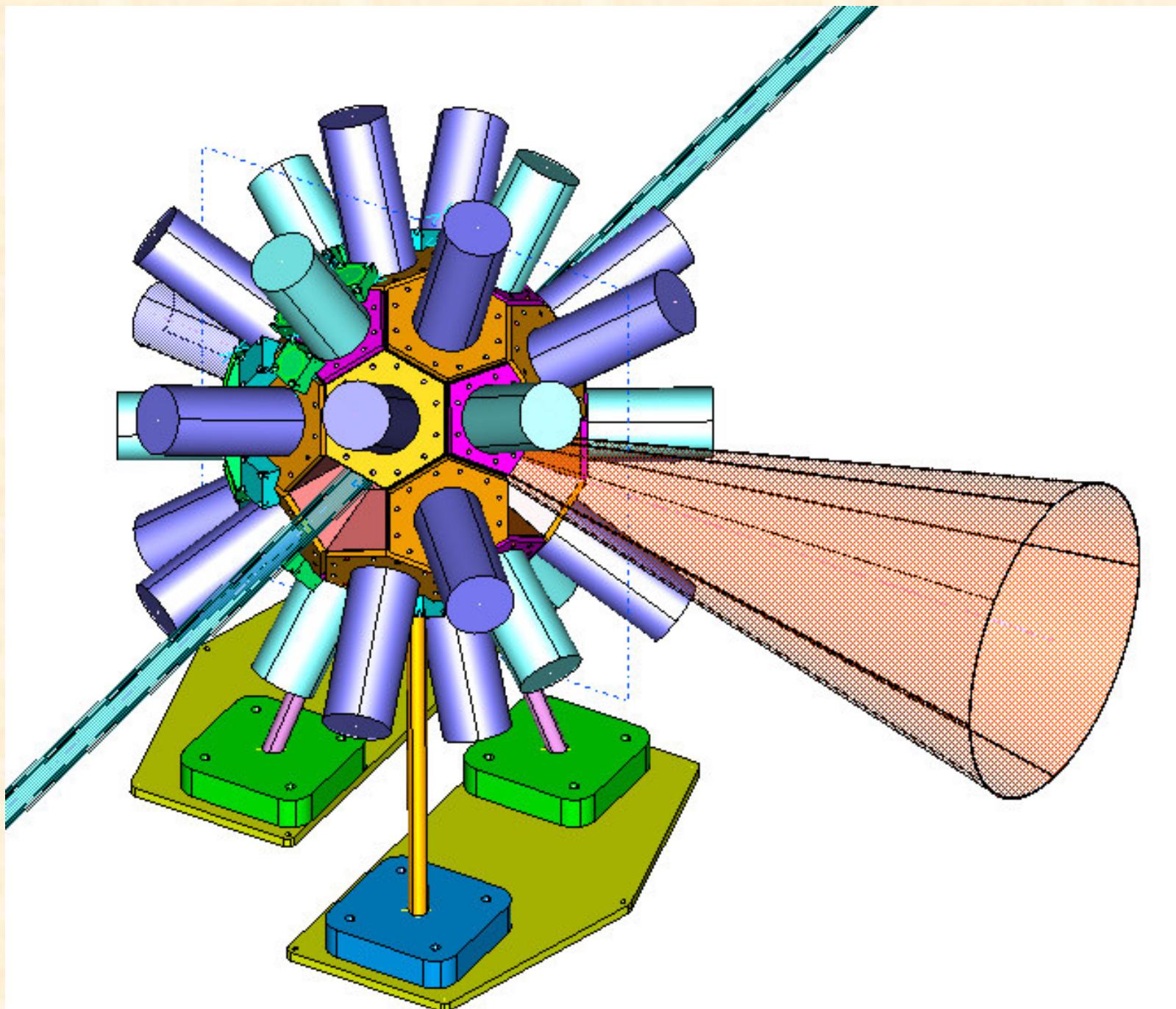
Length      76 mm  
Sides      44 mm  
              88 mm

PMT          3" dia XP3332/PB

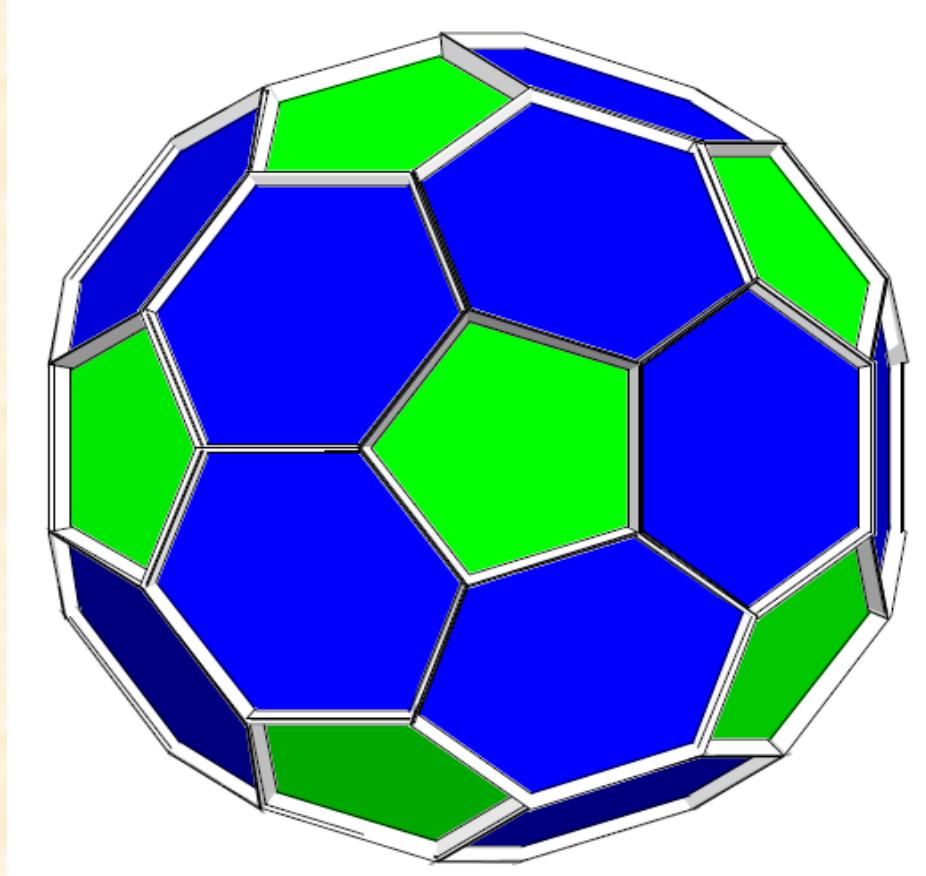
Energy  
Resolution ~6.5% @ 661 keV

Bias          +800 V

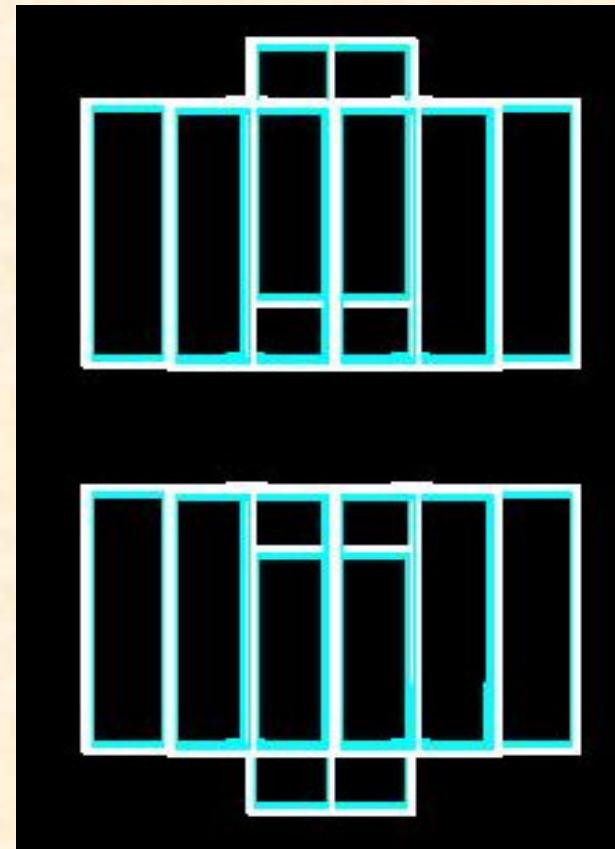




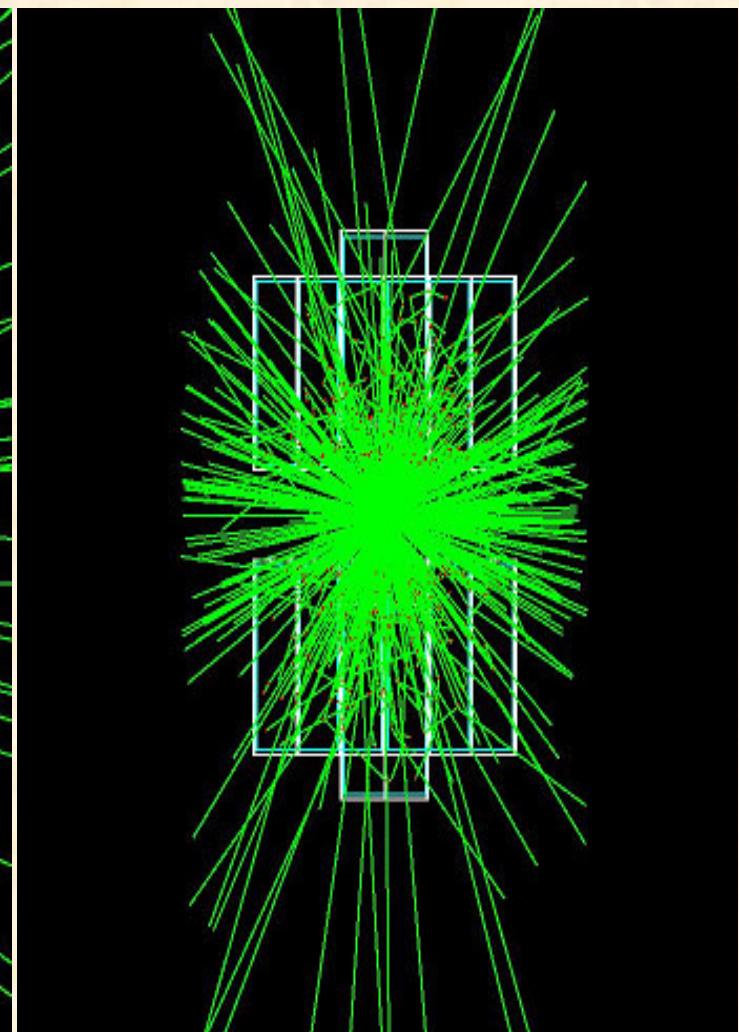
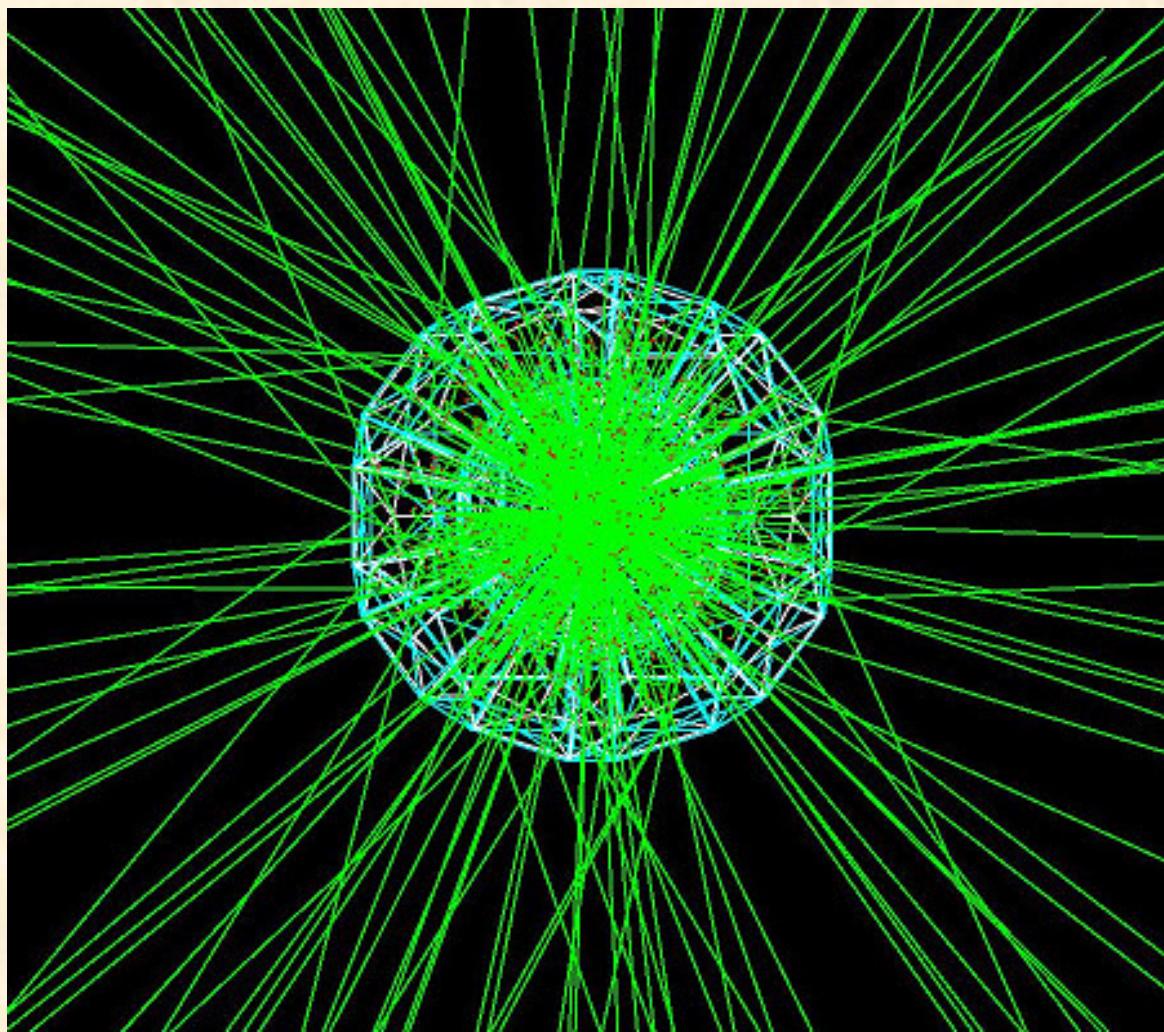
*32 detectors,  
Soccer ball geometry*



*14 detectors,  
castle geometry*



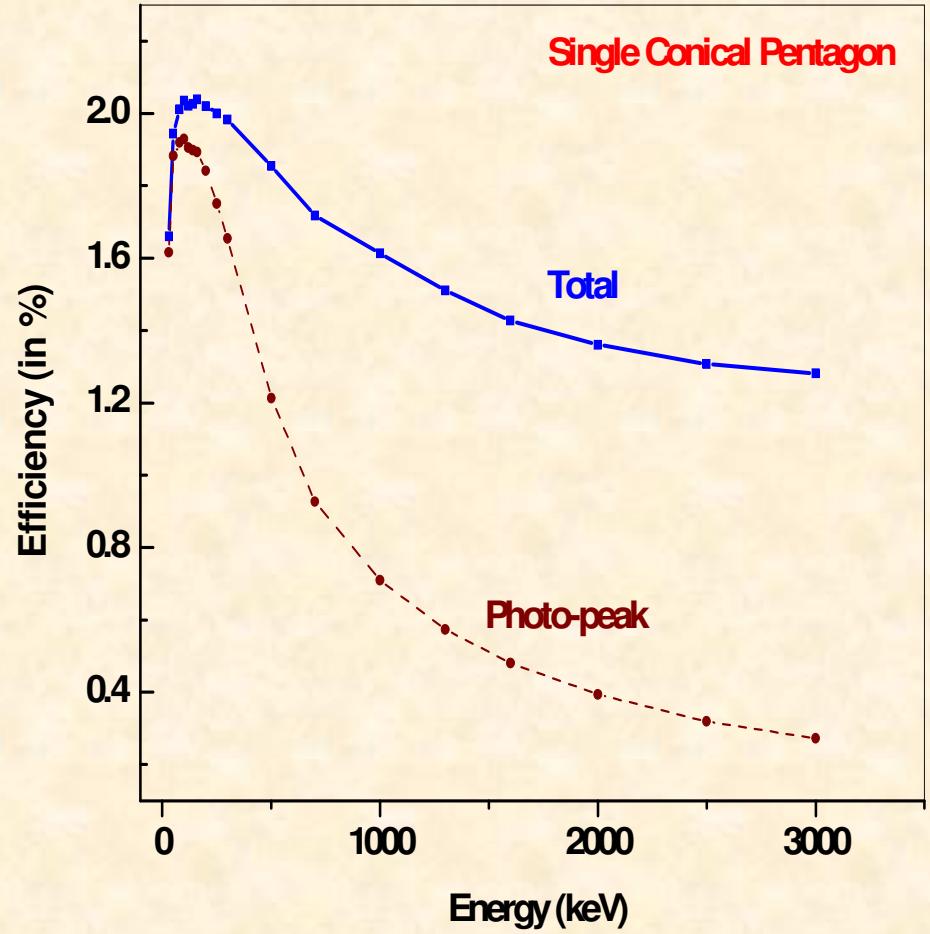
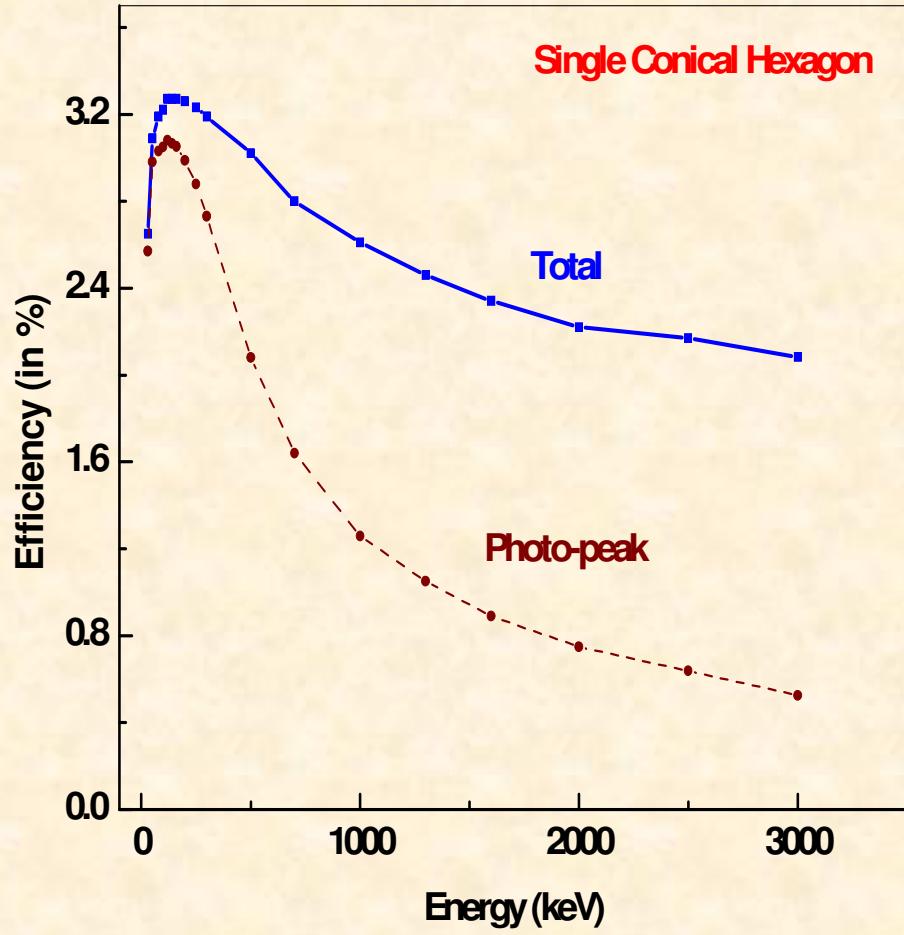
*Simulated diagrams using GEANT4*



**Simulated and measured efficiencies for a single detector  
for 662 keV at a distance of 10 cm from the centre of the  
face of the detector.**

Configuration	Absolute efficiency (in %)		Photo peak efficiency (in %)	
	GEANT4	Exp	GEANT4	Exp
Conical Hexagon (3")	$2.98 \pm 0.03$	$3.03 \pm 0.15$	$1.70 \pm 0.04$	$1.63 \pm 0.08$
Conical Pentagon (3")	$2.00 \pm 0.03$	$2.06 \pm 0.10$	$1.06 \pm 0.03$	$1.03 \pm 0.05$
Small Hexagon (4")	$1.03 \pm 0.06$	$1.08 \pm 0.05$	$0.47 \pm 0.03$	$0.47 \pm 3.90$

**(G. Anil Kumar, I. Mazumdar, D.A. Gothe, 2008 IEEE Nuclear Science  
Symposium conference record, N17-1, p. 1640, Dresden, Germany)**



## Simulated and measured efficiencies for different configurations of conical pentagons and hexagons

Configuration	Absolute efficiency (in %)		Photo peak efficiency (in %)	
	GEANT4	Exp	GEANT4	Exp
12 pentagons + 20 hexagons	$83.8 \pm 1.13$	$77.5 \pm 3.95$	$59.5 \pm 1.31$	$54.5 \pm 2.41$
10 pentagons + 20 hexagons	$79.4 \pm 1.10$	$76.8 \pm 3.84$	$49.0 \pm 1.23$	$46.5 \pm 2.32$
10 pentagons + 19 hexagons	$76.4 \pm 1.09$	$74.1 \pm 3.70$	$46.5 \pm 1.08$	$43.8 \pm 2.19$
9 pentagons + 20 hexagons	$77.5 \pm 1.06$	$75.0 \pm 3.75$	$47.5 \pm 1.38$	$45.2 \pm 2.26$
14 NaI system	$40.9 \pm 1.90$	$40.0 \pm 2.00$	$17.2 \pm 1.32$	$16.0 \pm 0.80$

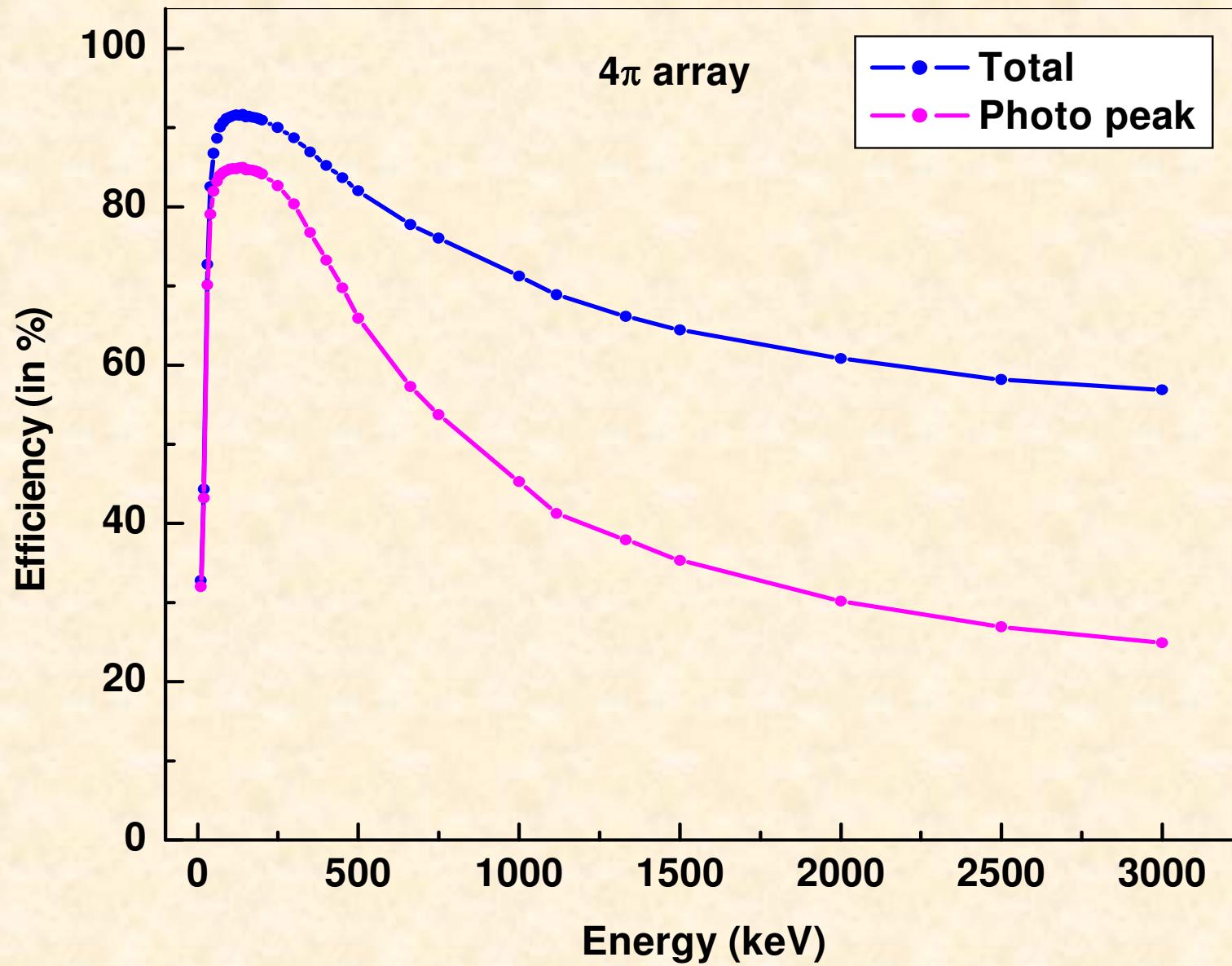
**Summary of results on simulated absolute efficiencies for full  $4\pi$  array and for the castle geometry of 14 elements for different mono energetic gamma rays**

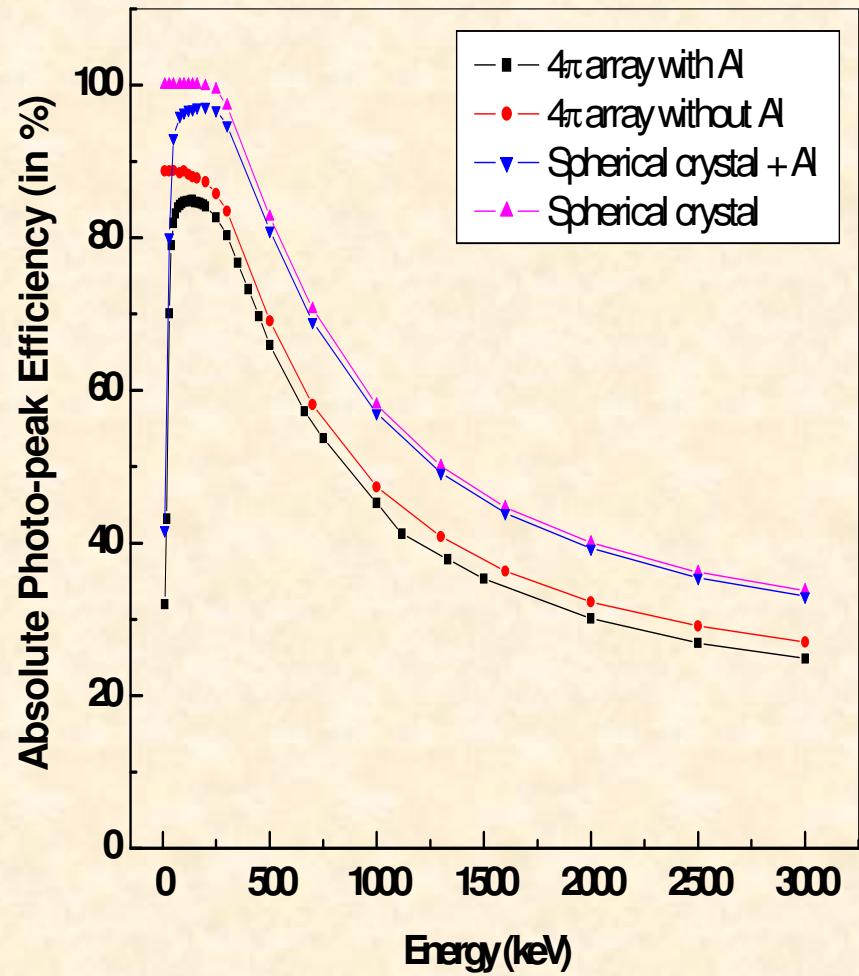
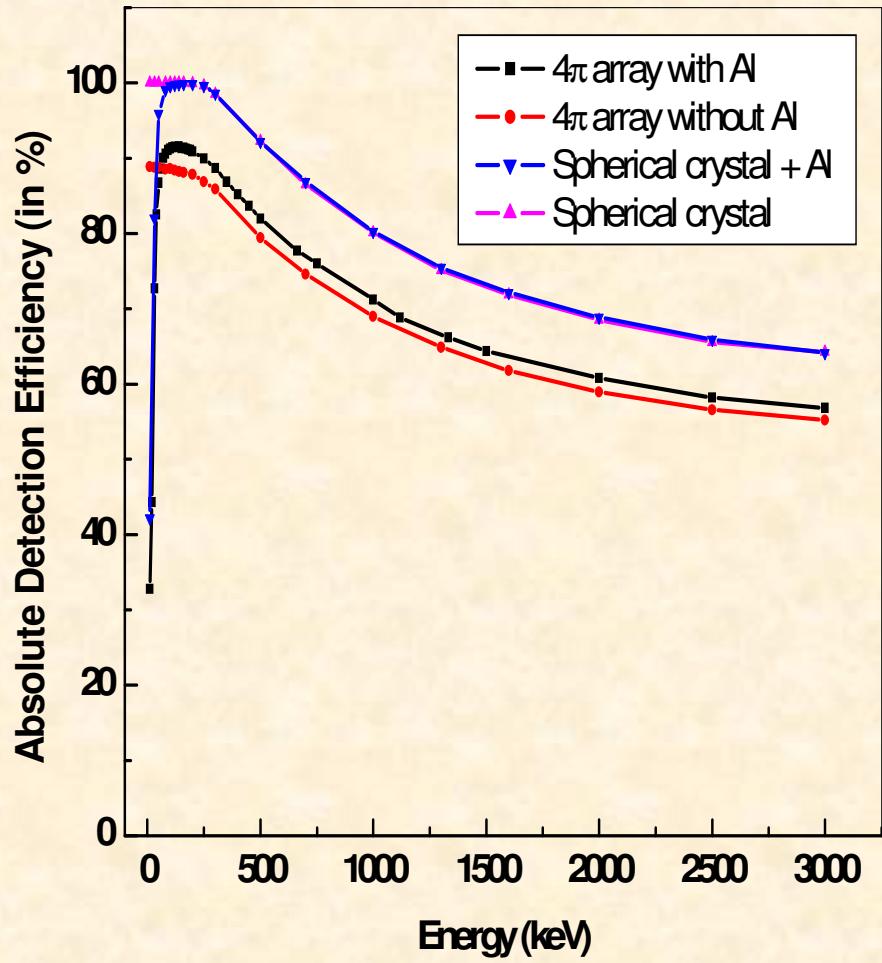
Energy (MeV)	Absolute efficiency (in %)		Photo peak efficiency (in %)	
	4 $\pi$ array	14 NaI	4 $\pi$ array	14 NaI
1	74.48 $\pm$ 1.16	37.09 $\pm$ 1.77	43.46 $\pm$ 0.96	13.50 $\pm$ 1.12
5	57.81 $\pm$ 1.23	29.65 $\pm$ 1.41	19.92 $\pm$ 0.72	4.10 $\pm$ 0.47
10	59.73 $\pm$ 0.82	32.00 $\pm$ 1.19	13.79 $\pm$ 0.66	2.00 $\pm$ 0.30
15	62.60 $\pm$ 1.30	33.93 $\pm$ 1.39	8.67 $\pm$ 0.56	0.96 $\pm$ 0.21
20	64.53 $\pm$ 0.88	35.28 $\pm$ 1.76	4.75 $\pm$ 0.32	0.41 $\pm$ 0.08
25	66.30 $\pm$ 1.16	36.43 $\pm$ 1.58	2.47 $\pm$ 0.18	0.17 $\pm$ 0.06
30	67.60 $\pm$ 1.18	37.15 $\pm$ 1.69	1.24 $\pm$ 0.08	0.06 $\pm$ 0.03
35	68.90 $\pm$ 1.11	37.98 $\pm$ 1.71	0.57 $\pm$ 0.14	0.02 $\pm$ 0.01
40	69.80 $\pm$ 1.07	37.46 $\pm$ 1.47	0.27 $\pm$ 0.03	0.009 $\pm$ 10 <sup>-3</sup>
45	70.54 $\pm$ 1.09	39.09 $\pm$ 1.87	0.12 $\pm$ 0.02	0.004 $\pm$ 10 <sup>-3</sup>
50	70.63 $\pm$ 1.13	39.38 $\pm$ 1.72	0.04 $\pm$ 0.01	0.001 $\pm$ 10 <sup>-4</sup>

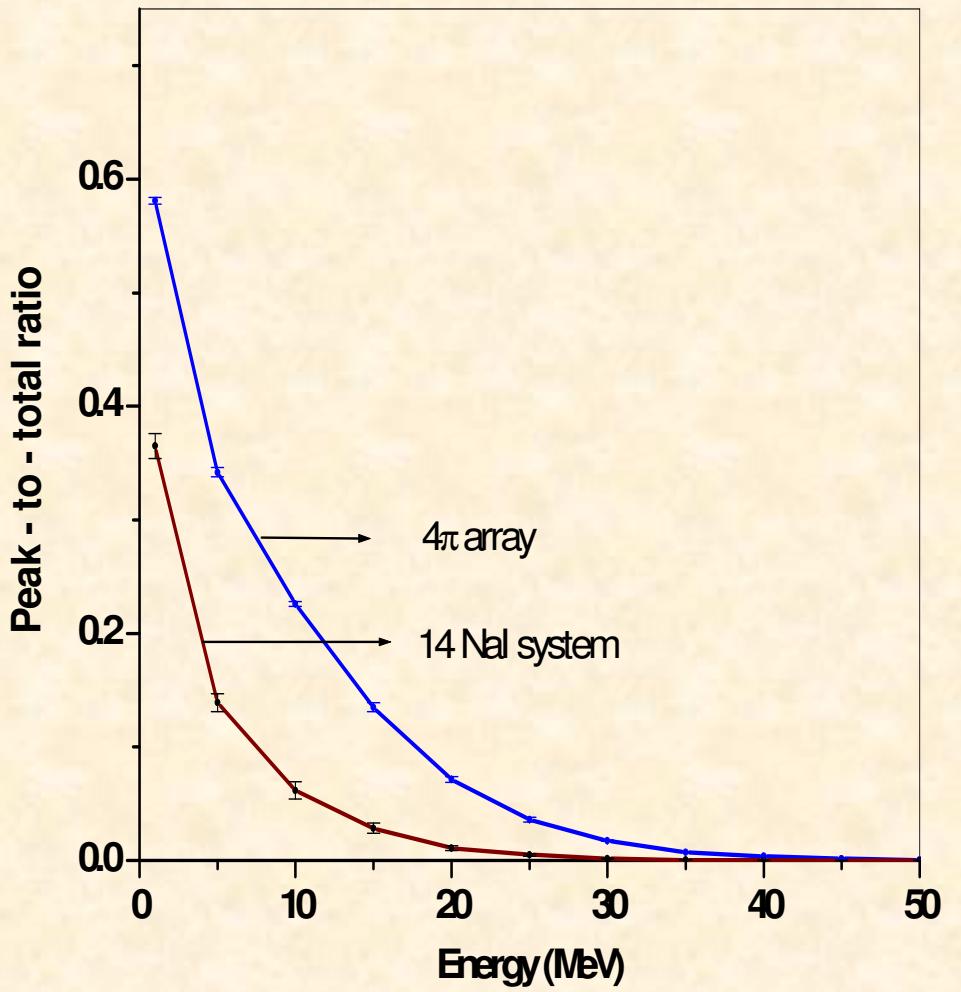
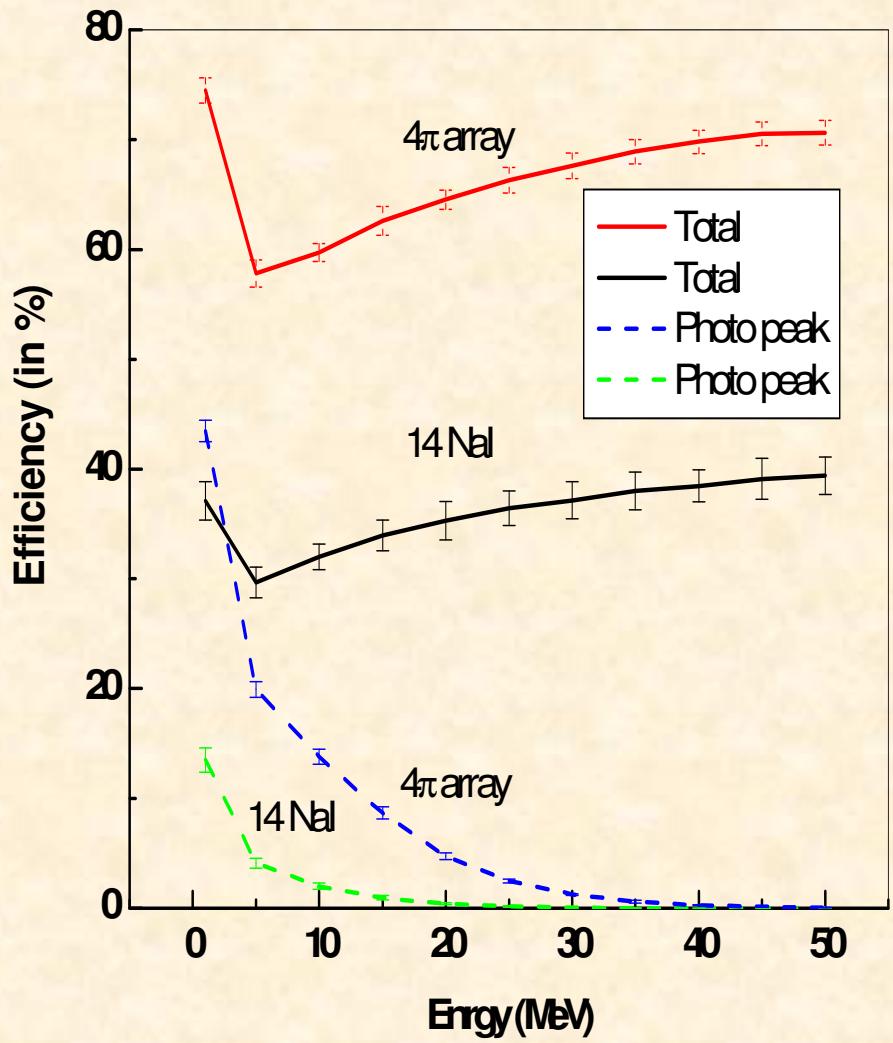
**Summary of results on simulated absolute efficiencies for full  $4\pi$  array of  $\text{LaBr}_3(\text{Ce})$  detectors for different mono energetic gamma rays**

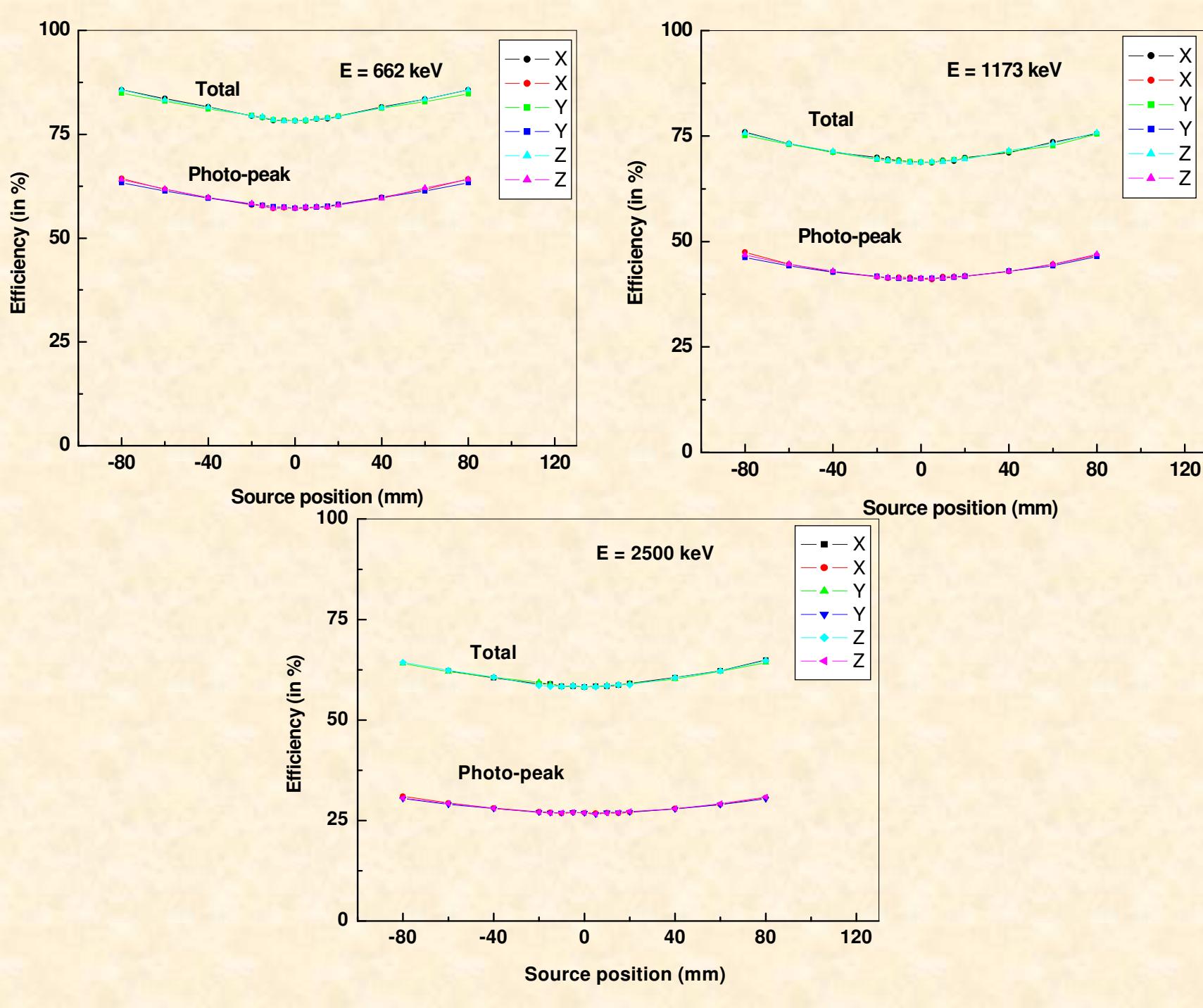
E (MeV)	Absolute Efficiency (in %)	Photo peak efficiency (in %)
0.662	90.00	71.00
1.173	82.60	55.63
1.332	81.00	52.75
5	68.42	32.63
10	71.25	25.64
15	72.70	18.38
20	75.03	11.37
30	78.31	4.41
40	79.73	1.56
50	81.56	0.55

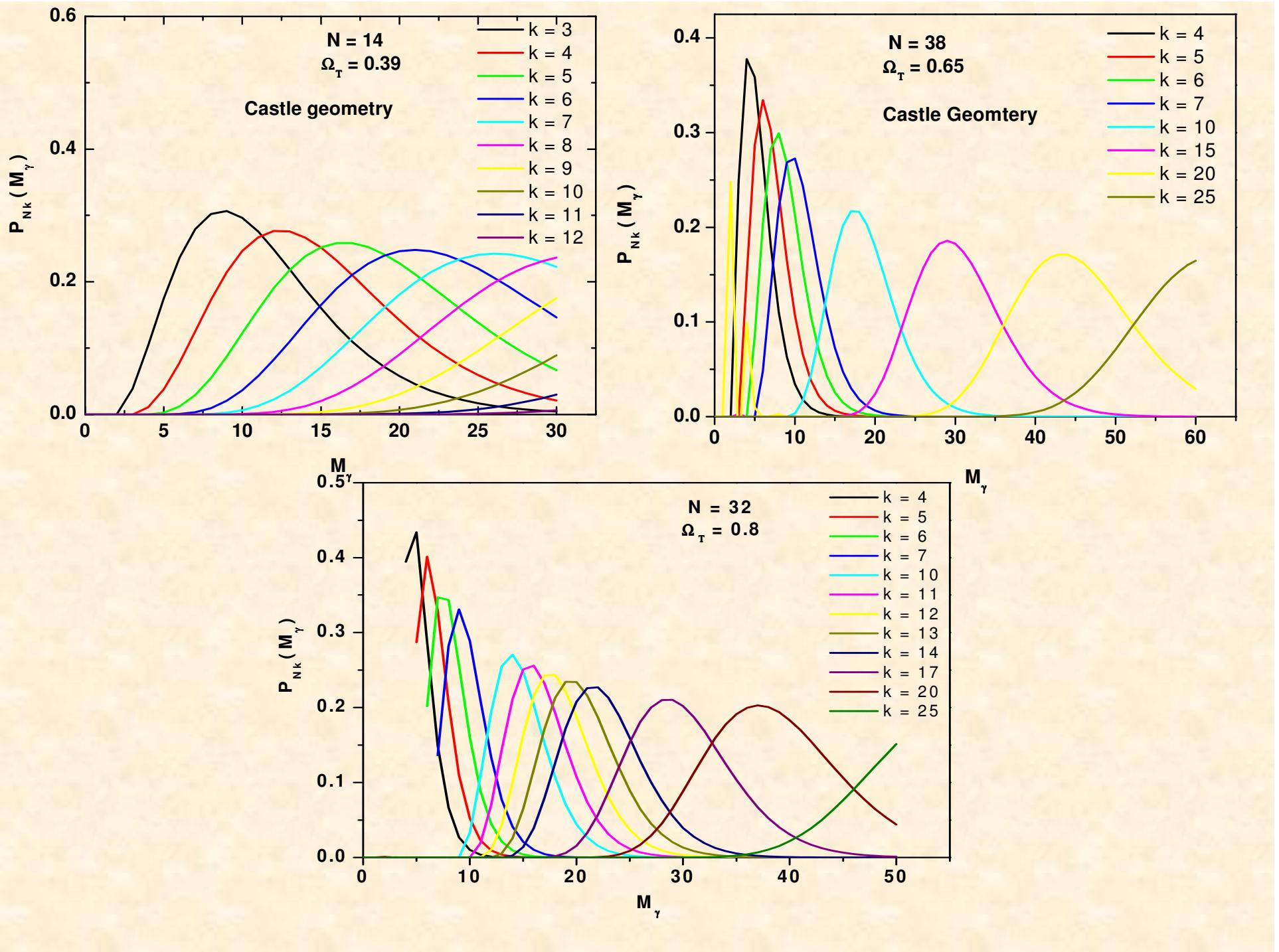
4pi-PARIS – 32 hexagons and pentagons (Dec. 2008) – by G.A. Kumar and I. Mazumdar  
<http://paris.ifj.edu.pl/documents/sim/>









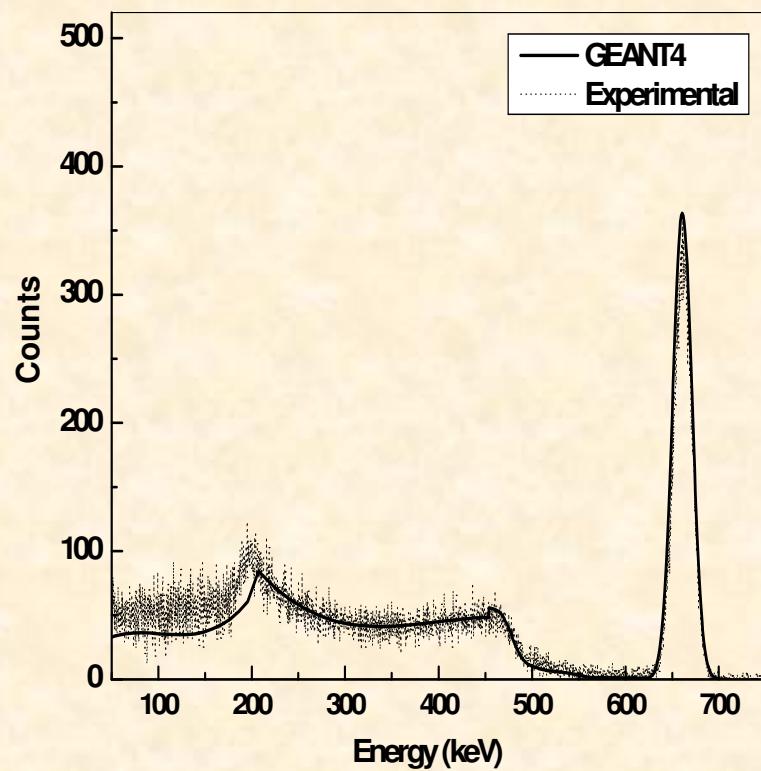


# Summary



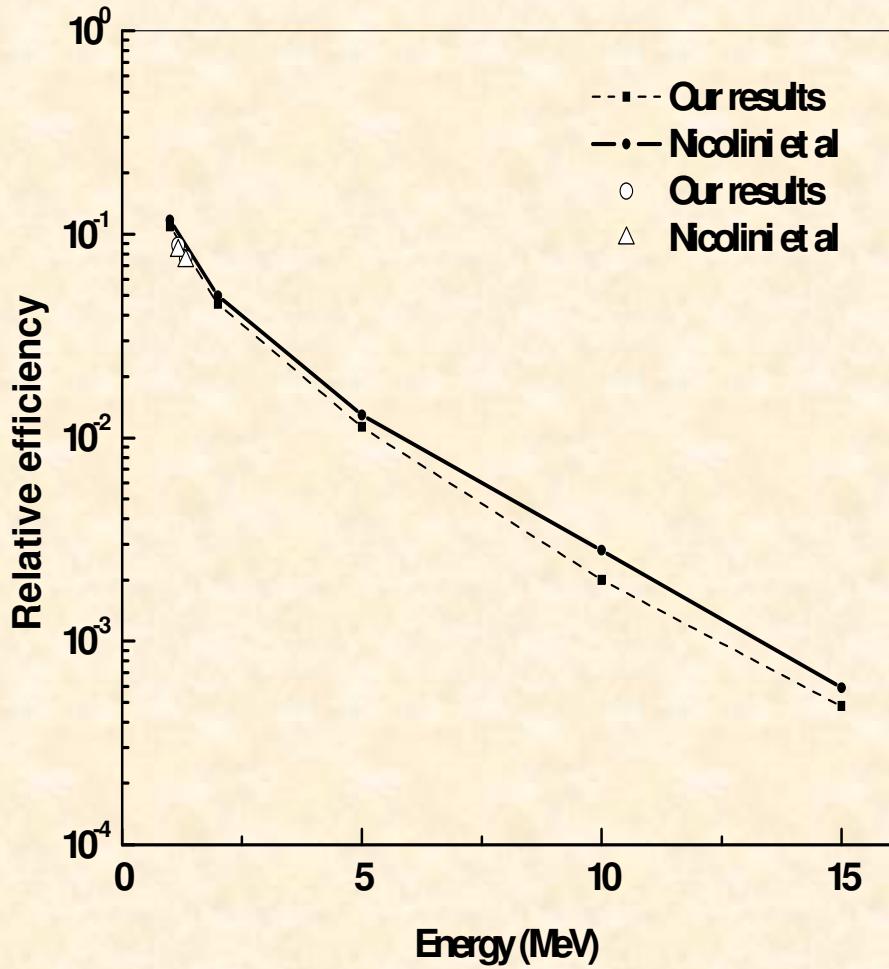
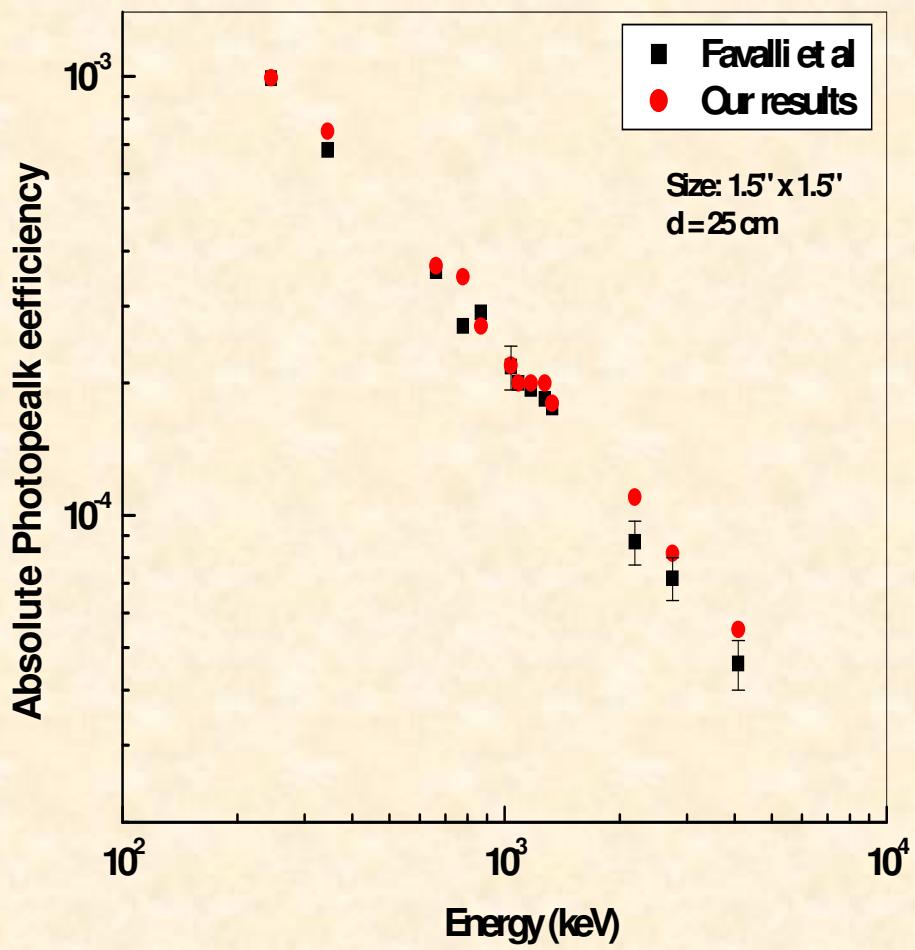
- Extensive simulations have been carried out for calculating the detection efficiencies of the individual detectors and the entire  $4\pi$  array.
- The total intrinsic and photo peak efficiencies have been measured accurately with calibrated low energy gamma ray sources and are found to be in very good agreement with the simulated results.
- Efficiency measurements and GEANT4 simulations have been carried out for a smaller array of 14 straight NaI(Tl) detectors of hexagonal cross sections packed in castle geometry and the results have been compared with the  $4\pi$  array.
- The fold distributions have also been calculated for different gamma multiplicities for both the 14 elements and the  $4\pi$  array.

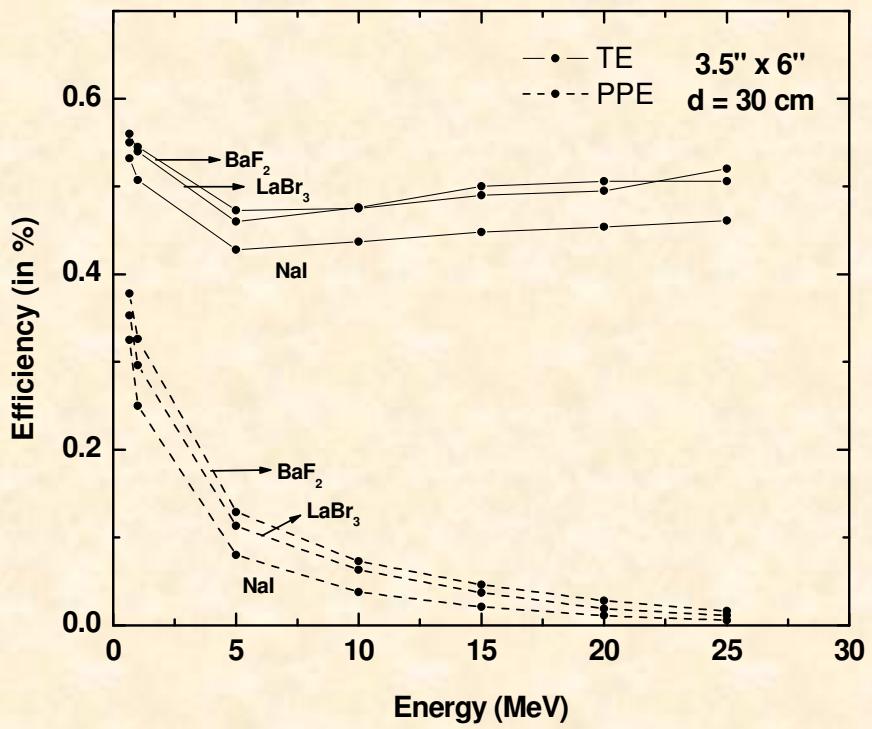
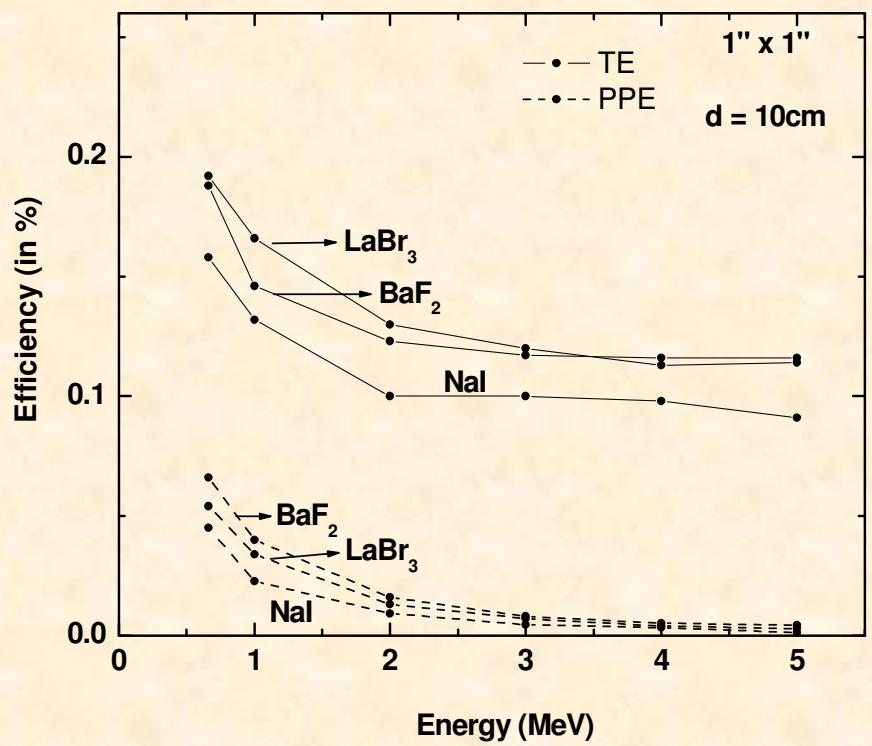
**A comparative study of efficiencies of  $\text{LaBr}_3(\text{Ce})$ ,  $\text{NaI}(\text{TI})$  and  $\text{BaF}_2$**

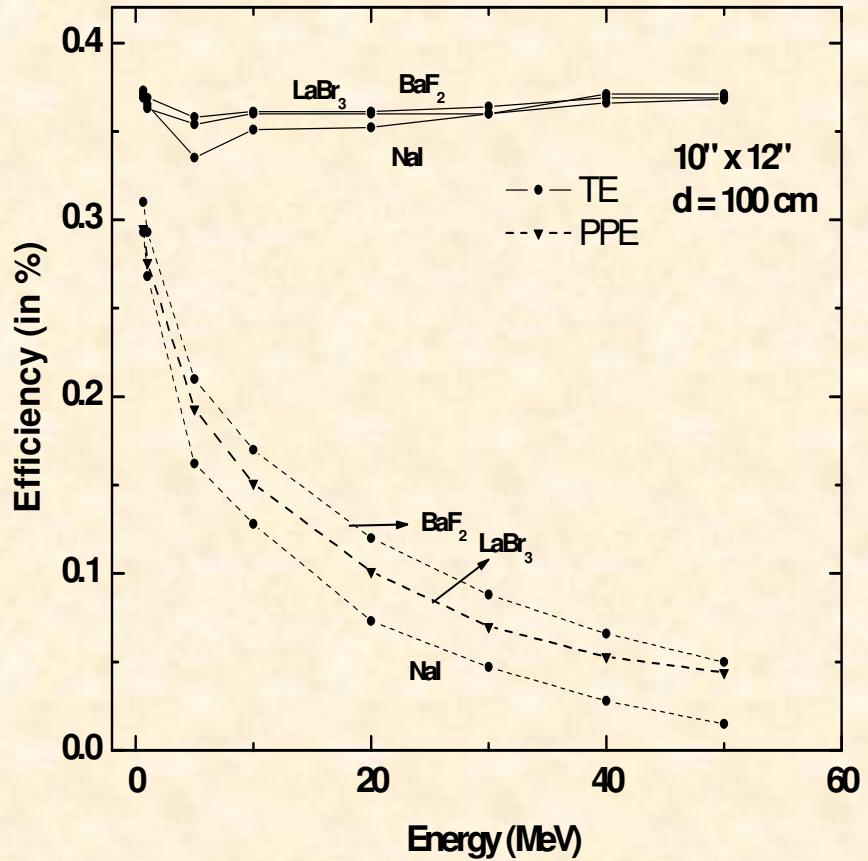
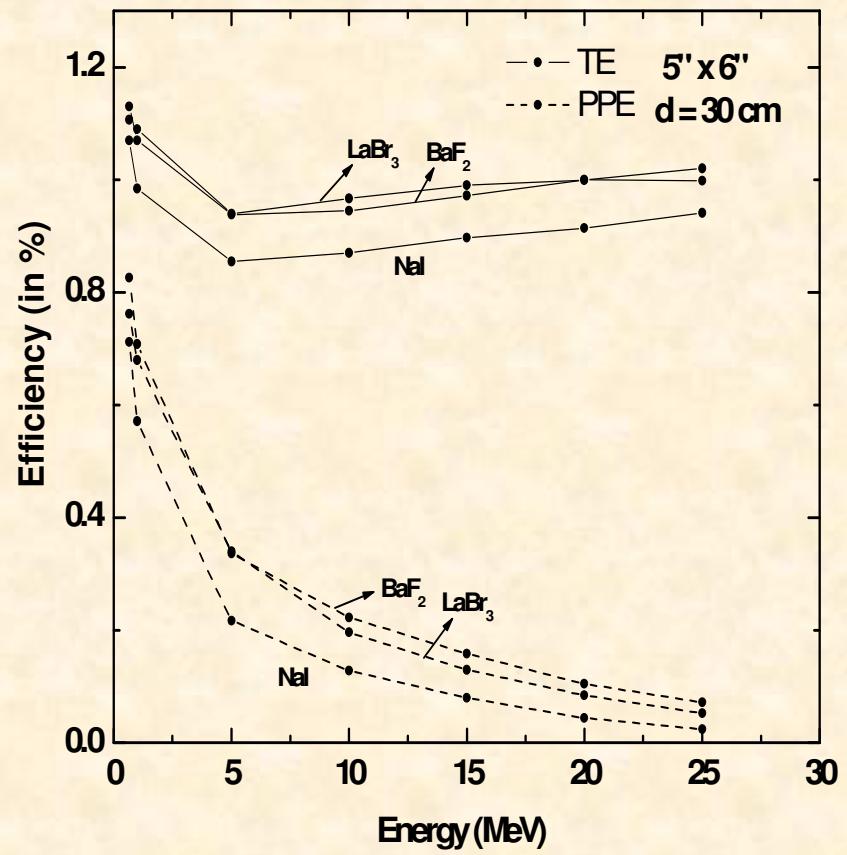


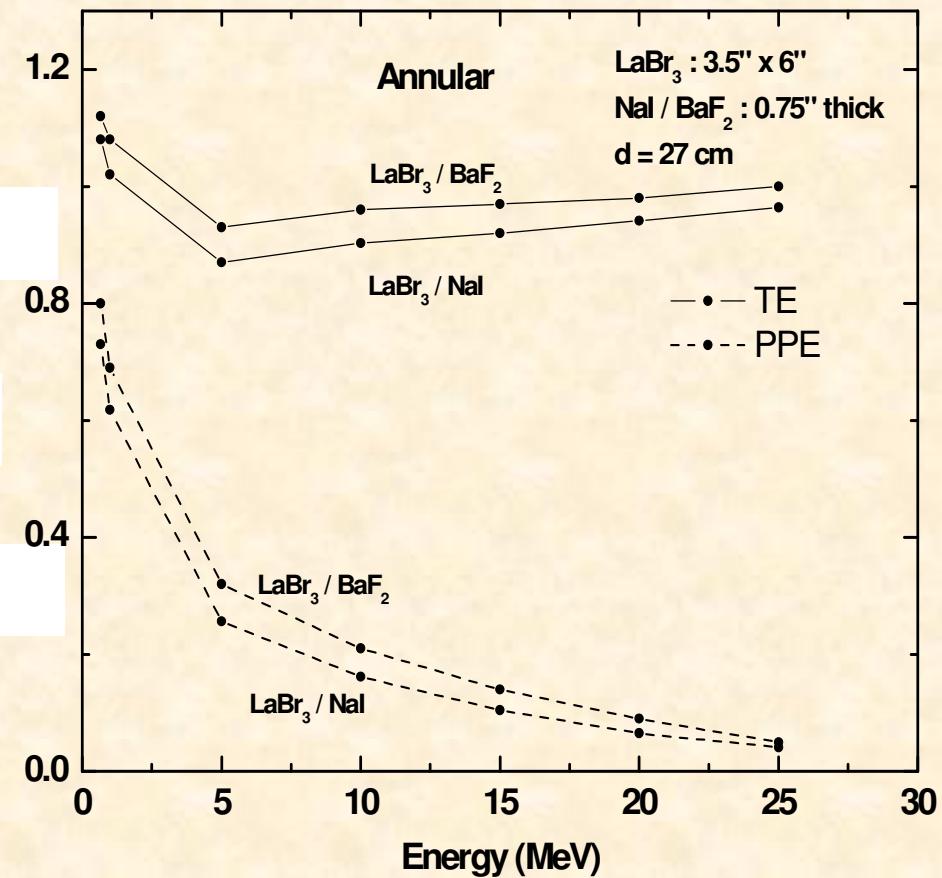
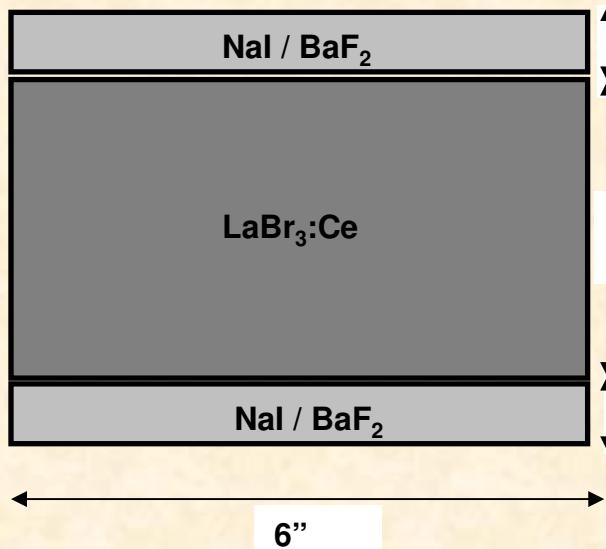
G. Anil Kumar, I. Mazumdar, D.A. Gothe,  
**Nucl. Instr. and Meth.A,**  
[doi:10.1016/j.nima.2009.08.075](https://doi.org/10.1016/j.nima.2009.08.075)

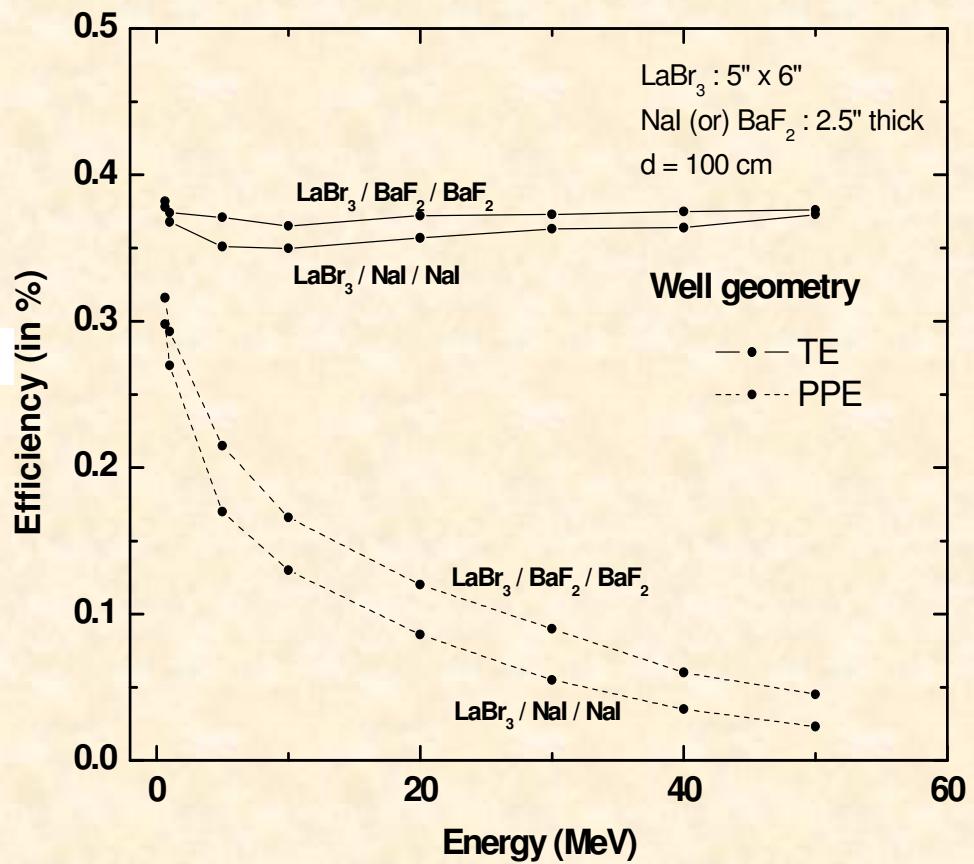
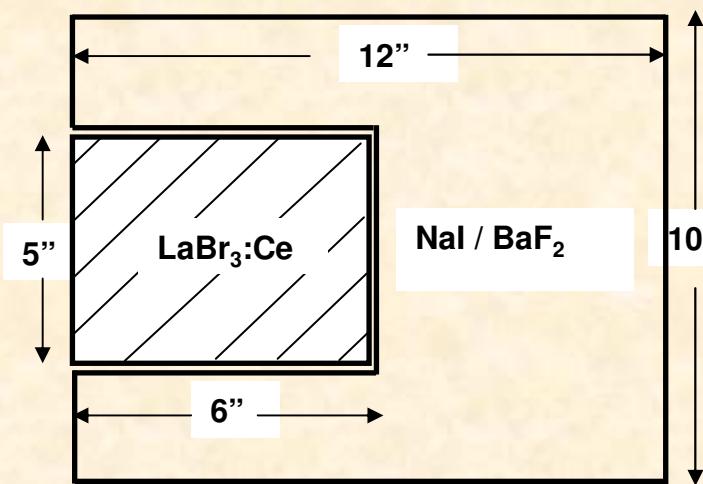
Distance (cm)	$\epsilon_{\text{total}}$		$\epsilon_{\text{peak}}$	
	GEANT4	Exp	GEANT4	Exp
15	<b>0.105 (0.012)</b>	<b>0.114 (0.005)</b>	<b>0.030 (0.004)</b>	<b>0.027 (0.001)</b>
25	<b>0.041 (0.003)</b>	<b>0.044 (0.002)</b>	<b>0.011 (0.001)</b>	<b>0.010 (0.001)</b>











## Summary

The simulation results are compared with recent measurements reported by other authors for 1" x 1" and for 1.5" x 1.5" detectors. A good agreement between simulations and measurements has been achieved

Simulation results are presented in connection with a proposition for high energy gamma detection using combination of LaBr<sub>3</sub>(Ce) with NaI(Tl)/BaF<sub>2</sub>.

**Close-geometry efficiency calibration and true coincidence  
summing correction**

## Calculation of coincidence summing correction factors

Assumptions:

- 1) Point source
- 2)  $\beta$ -radiation absorbed in the detector window

Count rate resulting from full absorption of  $\gamma$ -rays of energy  $E_1$  and  $E_2$  are given by

$$N_1 = A p_1 \epsilon_{p1}$$

$$N_2 = A p_2 \epsilon_{p2}$$

where

$p_1$  is emission probability of  $\gamma$ -rays with energy  $E_1$

$p_2$  is emission probability of  $\gamma$ -rays with energy  $E_2$

$\epsilon_{p1}$  is photo peak efficiency for gamma rays with energy  $E_1$

$\epsilon_{p2}$  is photo peak efficiency for gamma rays with energy  $E_2$

(T. Vidmar et al., NIM-A 508 (2003) 404)

The count rate in recorded full energy peak in the spectrum, however, is smaller. Since each  $\gamma_1$  is followed by a  $\gamma_2$  in coincidence, it may happen that both  $\gamma$ -rays are detected, thus leading to a single count. If the energy of  $\gamma_1$  is totally absorbed, this sum pulse is recorded at an energy between  $E_1$  and  $E_1 + E_2$  and the event is lost from the full energy peak of  $\gamma_1$

$$N'_1 = A p_1 \epsilon_{p1} - A p_1 \epsilon_{p1} \epsilon_{t2} = A p_1 \epsilon_{p1} (1 - w \epsilon_{t2})$$

where  $\epsilon_{t2}$  is total detection efficiency of  $\gamma_2$

Similarly,

$$N'_2 = A p_2 \epsilon_{p2} (1 - w \epsilon_{t1})$$

$$N'_{12} = A p_1 p_2 w \epsilon_{p1} \epsilon_{p2}$$

Three equations and four unknowns if the source is calibrated

If the source is non-calibrated point source,

$$A = N_{total} + \frac{N'_1 N'_2}{w N'_{12}}$$

Finally,

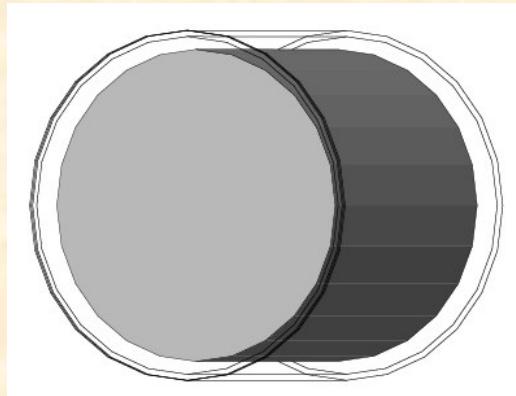
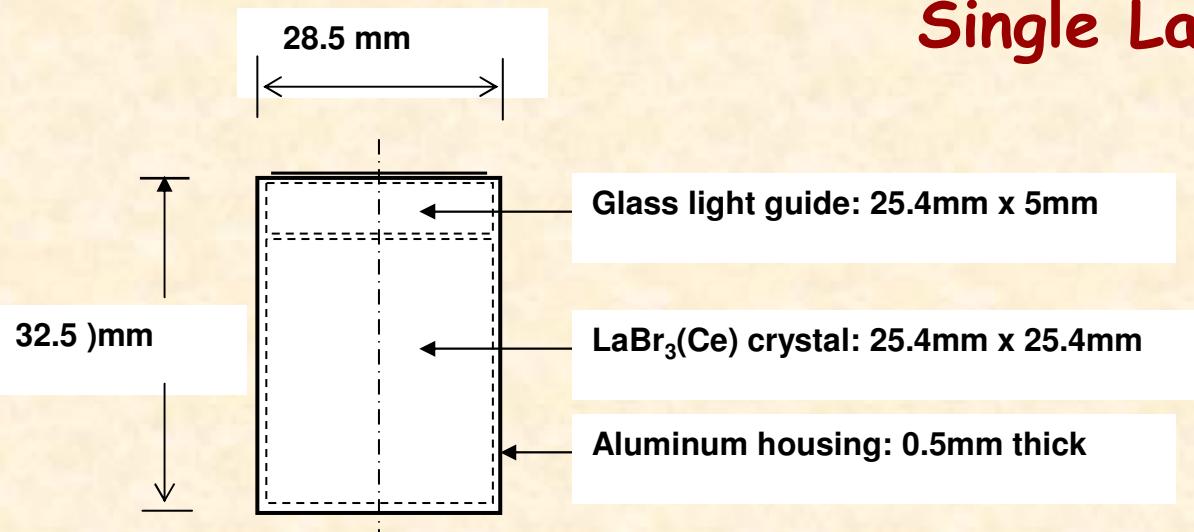
$$\epsilon_{p1} = \frac{\left( (N'_1 - N'_2) + \sqrt{(N'_1 - N'_2)^2 + 4AN'_{12}} \right)}{2A} + \sqrt{\frac{N'_1 N'_{12}}{AN'_2}}$$

$$\epsilon_{p2} = \frac{N'_{12}}{A \epsilon_{p1}}$$

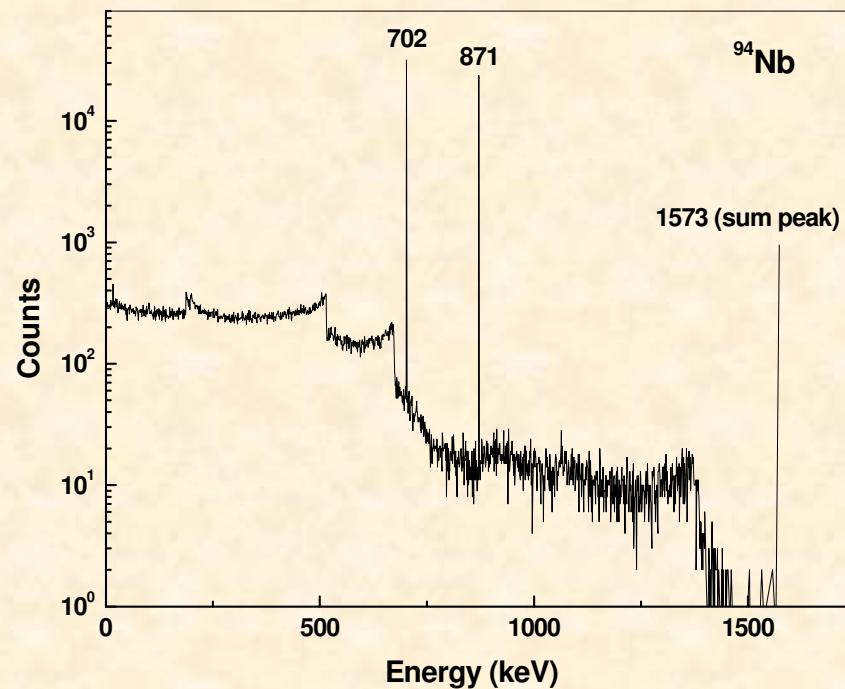
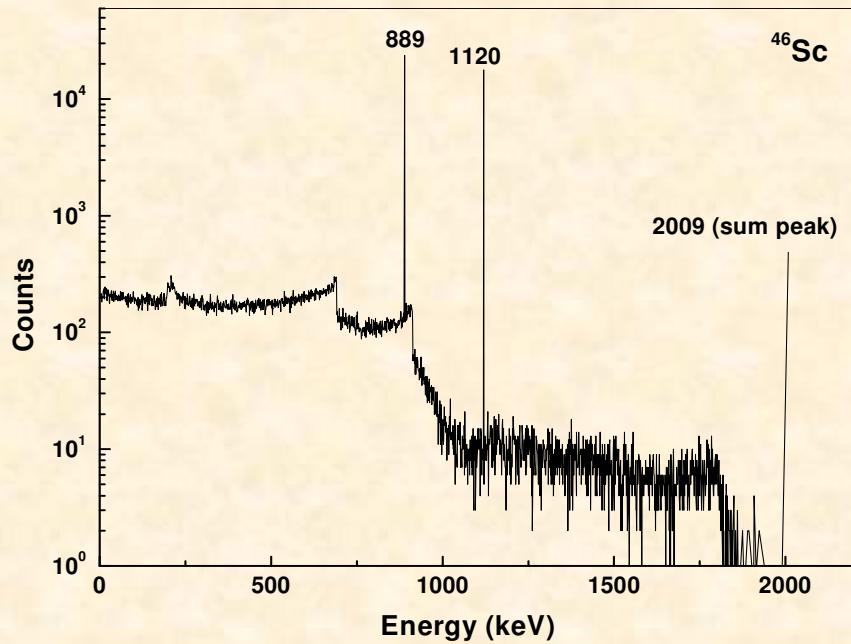
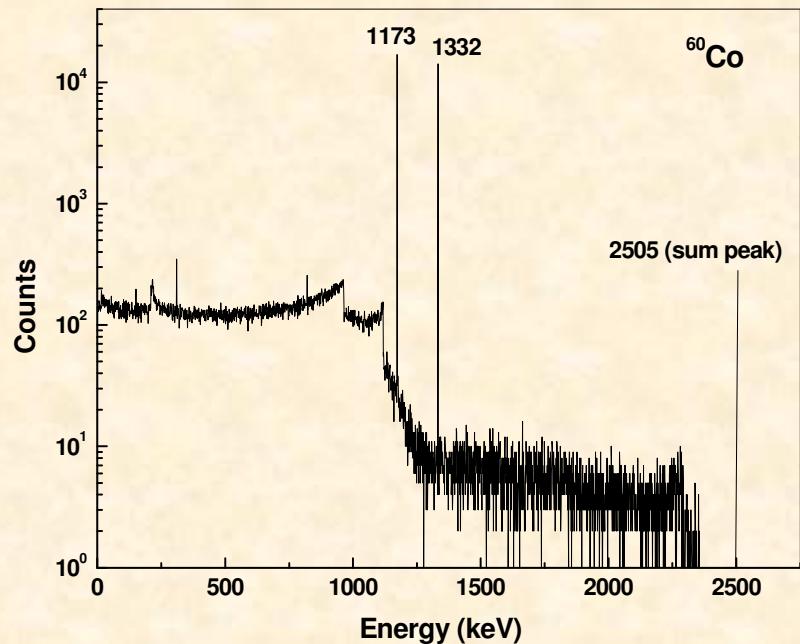
$$\epsilon_{t1} = 1 - \frac{\epsilon_{p1} N'_2}{N'_{12}}$$

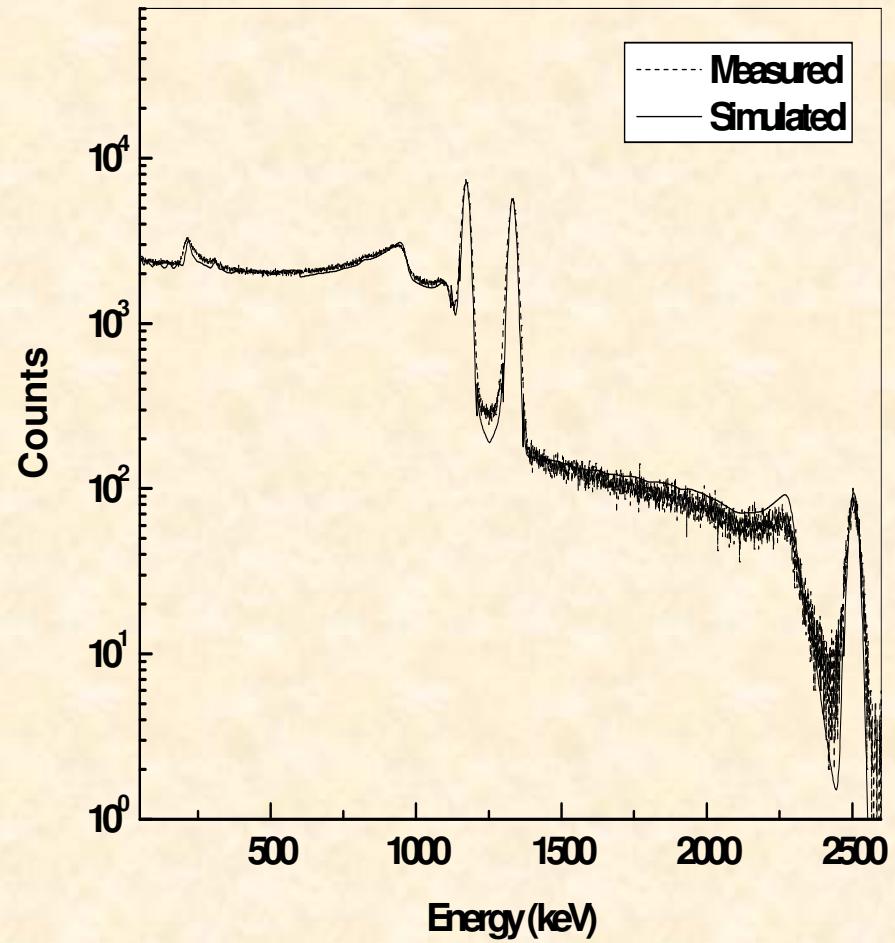
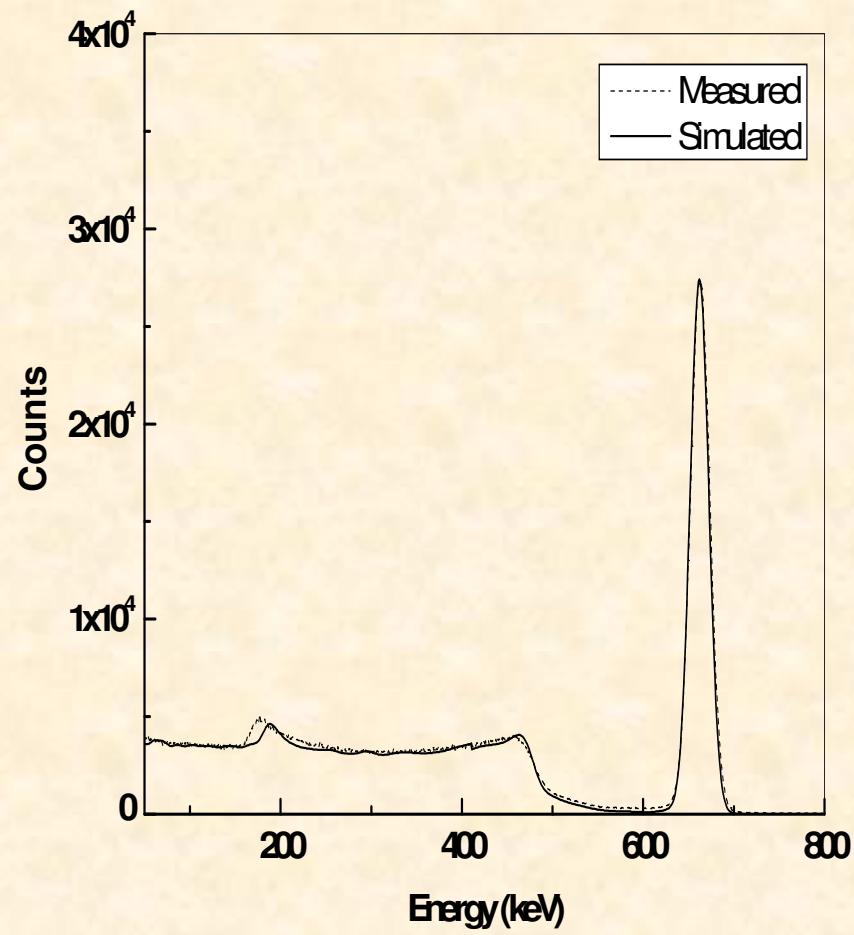
$$\epsilon_{t2} = 1 - \frac{N'_1}{A \epsilon_{p1}}$$

## Single LaBr<sub>3</sub>(Ce) detector



Nuclide	E <sub>1</sub> (keV)	E <sub>2</sub> (keV)	b <sub>1</sub> (%)	b <sub>2</sub> (%)
<sup>60</sup> Co	1173.23	1332.50	99.85	99.98
<sup>46</sup> Sc	889.27	1120.54	99.98	99.98
<sup>94</sup> Nb	702.64	871.11	99.79	99.86
<sup>24</sup> Na	1368.63	2754.03	99.99	99.85





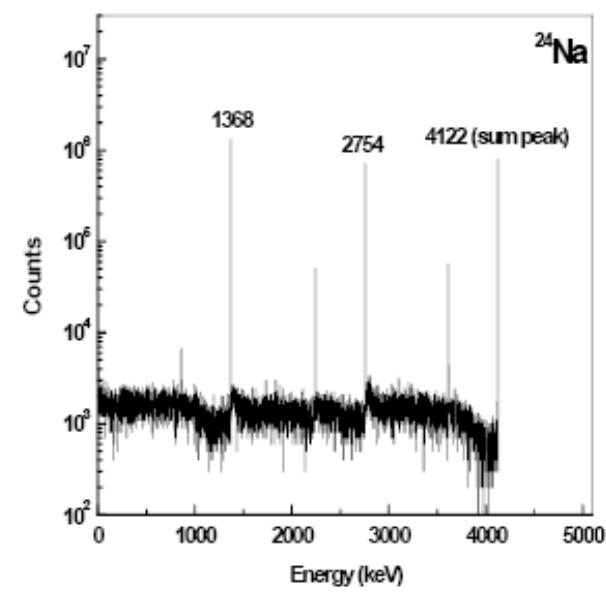
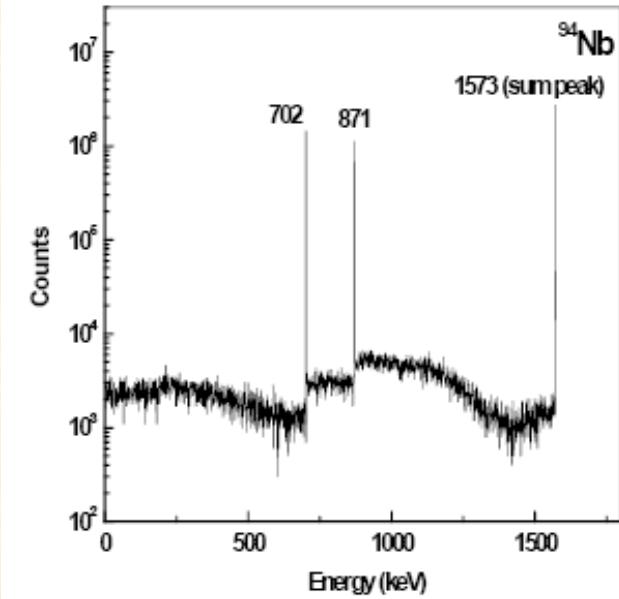
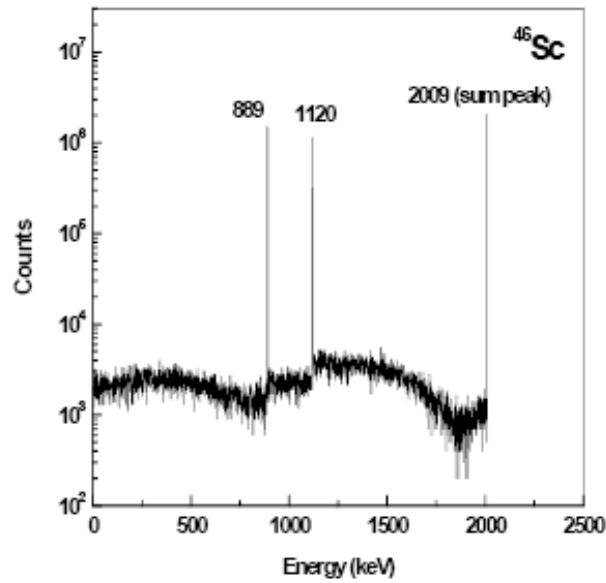
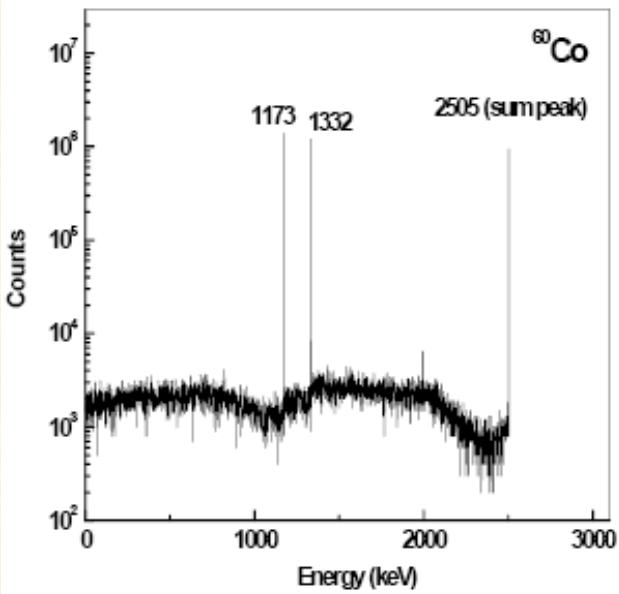
## Simulated true efficiencies for double photon emitters ( $^{60}\text{Co}$ , $^{46}\text{Sc}$ and $^{94}\text{Nb}$ ) and for mono-energetic gamma sources of similar energies

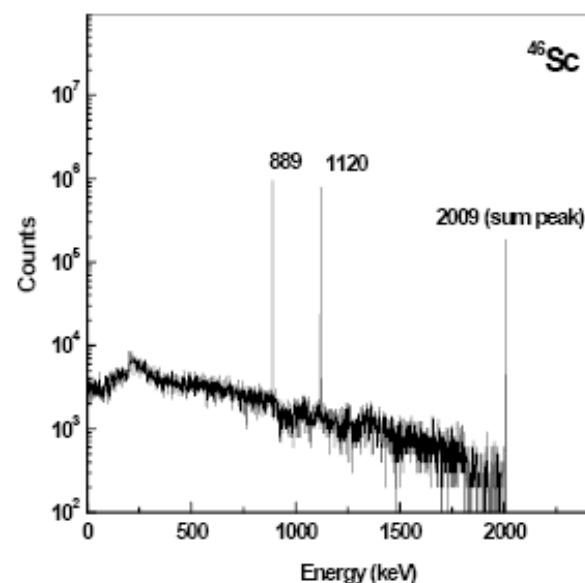
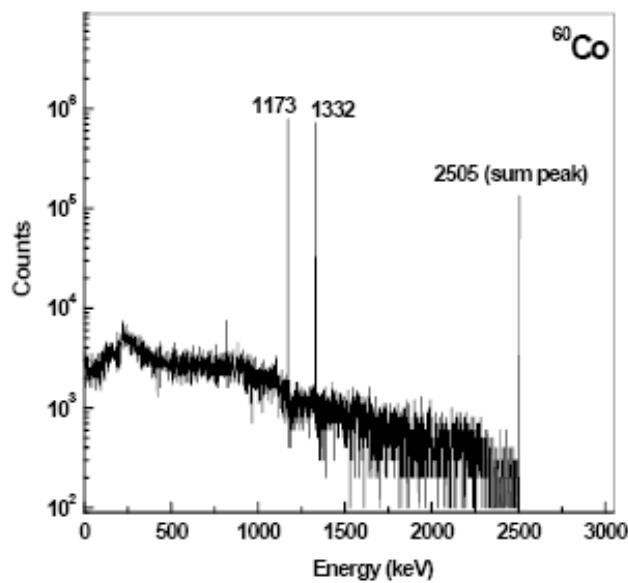
Source	$\varepsilon_{t1}$ (%)	$\varepsilon_{t2}$ (%)	$\varepsilon_{p1}$ (%)	$\varepsilon_{p2}$ (%)
$^{60}\text{Co}$	12.63 (1.10)	11.75 (1.14)	1.87 (0.11)	1.60 (0.10)
Mono energetic	10.80 (0.77)	9.74 (1.00)	1.84 (0.16)	1.56 (0.14)
$^{46}\text{Sc}$	12.67 (0.40)	11.75 (0.46)	2.67 (0.10)	1.97 (0.07)
Mono energetic	11.40 (0.87)	10.40 (0.79)	2.60 (0.23)	1.94 (0.17)
$^{94}\text{Nb}$	12.67 (0.46)	11.00 (0.40)	3.61 (0.09)	2.74 (0.07)
Mono energetic	12.58 (0.95)	11.47 (0.90)	3.64 (0.30)	2.71 (0.24)

Nuclide	Energy (keV)	Coincidence summing correction factor
$^{60}\text{Co}$	1173.23	0.882 (0.075)
	1332.50	0.873 (0.077)
$^{46}\text{Sc}$	889.27	0.889 (0.076)
	1120.54	0.873 (0.077)
$^{94}\text{Nb}$	702.62	0.877 (0.037)
	871.1	0.860 (0.041)

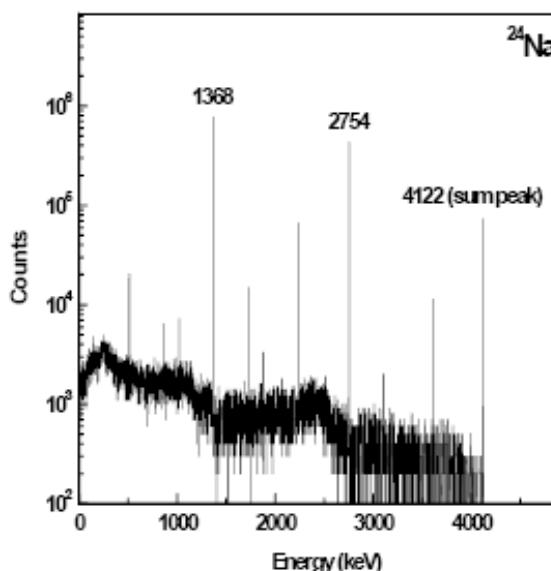
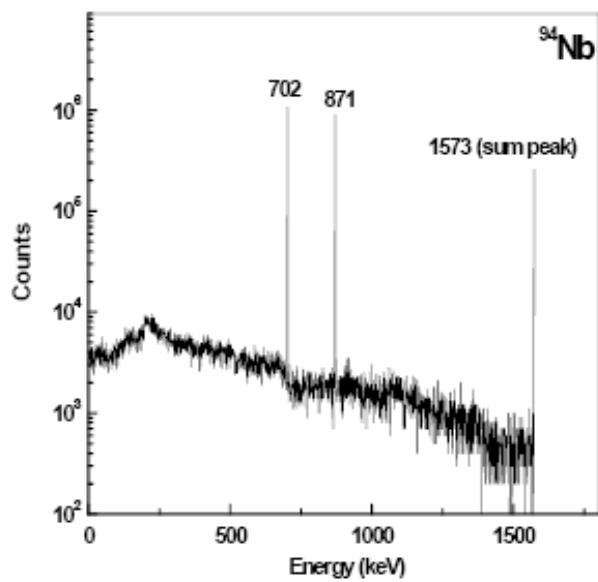
Nuclide	Energy (keV)	Total efficiency (%)		Photo-peak efficiency (%)	
		Measured	Simulated	Measured	Simulated
<sup>137</sup> Cs	<b>661.6</b>	<b>12.01 (0.60)</b>	<b>12.90 (1.00)</b>	<b>3.41 (0.17)</b>	<b>3.93 (0.34)</b>
<sup>60</sup> Co	<b>1173.23</b>	<b>14.39 (2.19)</b>	<b>12.63 (1.10)</b>	<b>1.65 (0.09)</b>	<b>1.87 (0.11)</b>
	<b>1332.50</b>	<b>13.94 (2.20)</b>	<b>11.75 (1.14)</b>	<b>1.51 (0.08)</b>	<b>1.60 (0.10)</b>

Energy (keV)	Coincidence summing correction factor	
	Experimental	Simulated
<b>1173.23</b>	<b>0.860 (0.048)</b>	<b>0.882 (0.075)</b>
<b>1332.50</b>	<b>0.856 (0.048)</b>	<b>0.883 (0.077)</b>





Castle geometry

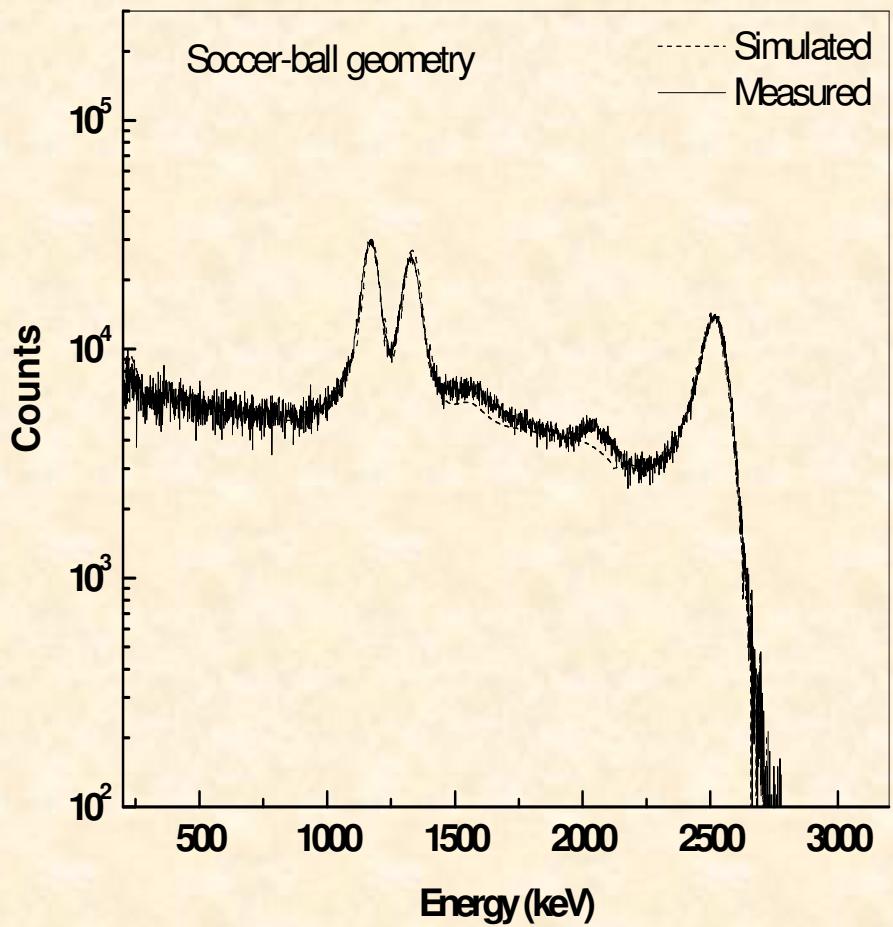
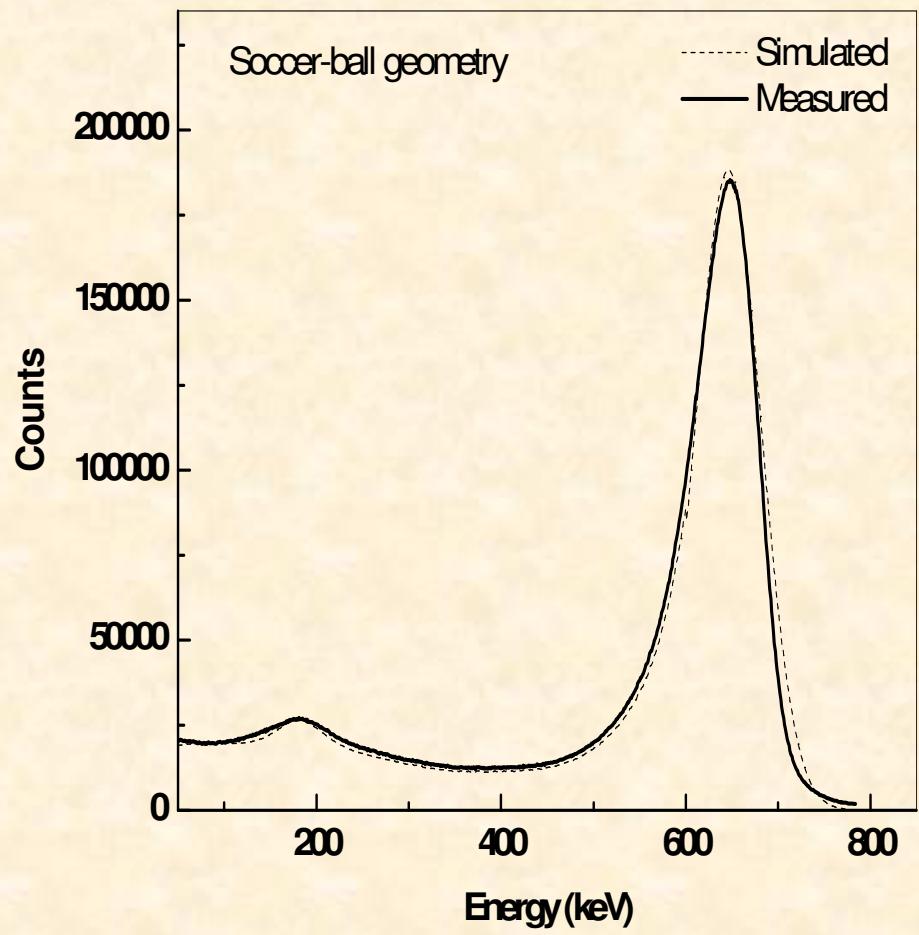


## Soccer-ball geometry

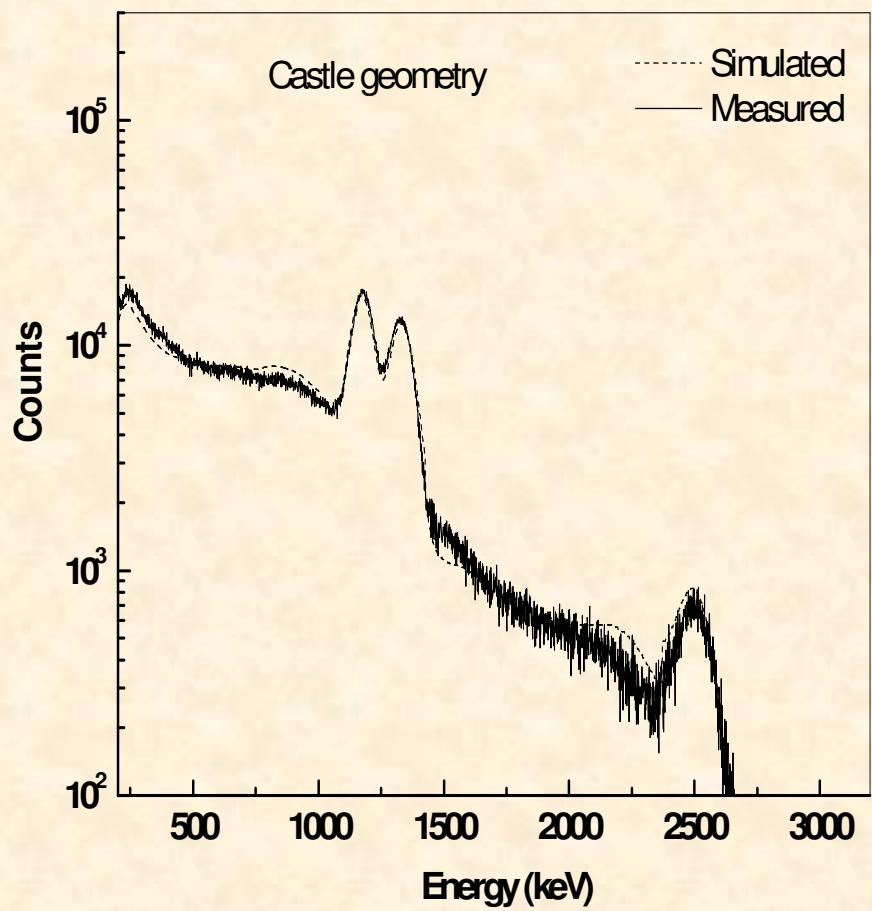
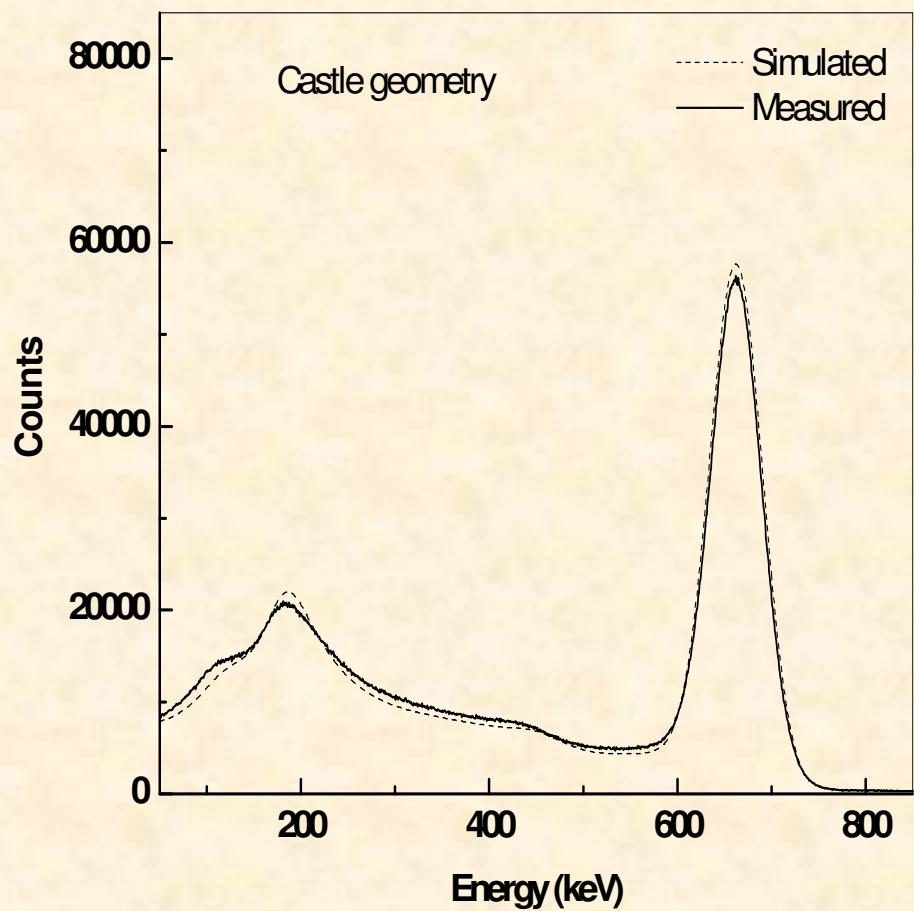
Source	$\varepsilon_{t1}$ (%)	$\varepsilon_{t2}$ (%)	$\varepsilon_{p1}$ (%)	$\varepsilon_{p2}$ (%)
$^{60}\text{Co}$	68.2 (1.1)	66.7 (1.2)	41.2 (1.4)	37.7 (1.3)
Mono energetic	68.5 (1.3)	66.2 (1.1)	41.0 (1.3)	38.1 (1.0)
$^{46}\text{Sc}$	72.6 (1.1)	70.0 (1.2)	49.5 (2.0)	41.5 (1.6)
Mono energetic	73.1 (1.5)	69.2 (1.3)	48.4 (1.5)	42.2 (1.3)
$^{94}\text{Nb}$	76.2 (1.1)	73.9 (1.3)	56.2 (2.7)	48.1 (2.3)
Mono energetic	76.9 (1.2)	73.6 (1.2)	55.5 (1.3)	49.2 (1.4)
$^{24}\text{Na}$	68.4 (2.0)	61.9 (2.4)	35.0 (2.2)	23.0 (1.4)
Mono energetic	66.0 (1.2)	57.4 (1.0)	37.5 (1.2)	25.7 (0.8)

## Castle geometry

Source	$\varepsilon_{t1}$ (%)	$\varepsilon_{t2}$ (%)	$\varepsilon_{p1}$ (%)	$\varepsilon_{p2}$ (%)
$^{60}\text{Co}$	36.6 (1.9)	35.2 (1.9)	12.4 (0.8)	11.3 (0.7)
Mono energetic	35.7 (1.9)	34.4 (1.7)	12.2 (1.1)	11.1 (1.0)
$^{46}\text{Sc}$	38.8 (1.9)	36.8 (1.1)	14.7 (0.9)	12.8 (0.8)
Mono energetic	38.2 (1.8)	36.0 (1.9)	14.6 (1.2)	12.6 (1.1)
$^{94}\text{Nb}$	40.5 (1.9)	38.4 (2.0)	17.2 (0.8)	14.9 (0.7)
Mono energetic	40.6 (1.0)	38.7 (1.8)	17.3 (1.4)	15.0 (1.1)
$^{24}\text{Na}$	36.3 (1.6)	30.4 (2.0)	10.9 (0.6)	6.9 (0.4)
Mono energetic	34.2 (1.9)	30.3 (1.6)	11.0 (1.0)	6.7 (0.6)



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## Soccer-ball geometry

Nuclide	Energy (keV)	Total efficiency (%)		Photo-peak efficiency (%)	
		Measured	Simulated	Measured	Simulated
$^{137}\text{Cs}$	<b>661.6</b>	<b>77.7 (3.9)</b>	<b>78.2 (1.1)</b>	<b>55.3 (2.4)</b>	<b>57.7 (1.3)</b>
$^{60}\text{Co}$	<b>1173.23</b>	<b>71.2 (1.3)</b>	<b>68.2 (1.1)</b>	<b>38.4 (1.6)</b>	<b>40.2 (1.5)</b>
	<b>1332.50</b>	<b>68.3 (1.4)</b>	<b>66.7 (1.2)</b>	<b>35.9 (1.3)</b>	<b>37.7 (1.3)</b>

## Castle geometry

Nuclide	Energy (keV)	Total efficiency (%)		Photo-peak efficiency (%)	
		Measured	Simulated	Measured	Simulated
$^{137}\text{Cs}$	<b>661.6</b>	<b>40.0 (2.0)</b>	<b>40.9 (1.9)</b>	<b>16.4 (0.8)</b>	<b>17.2 (1.3)</b>
$^{60}\text{Co}$	<b>1173.23</b>	<b>38.4 (1.9)</b>	<b>36.6 (1.9)</b>	<b>11.8 (1.6)</b>	<b>12.4 (0.8)</b>
	<b>1332.50</b>	<b>37.1 (2.0)</b>	<b>35.2 (1.9)</b>	<b>10.9 (1.8)</b>	<b>11.3 (0.7)</b>

## Experimental and simulated coincidence summing correction factors for both the arrays for $^{60}\text{Co}$

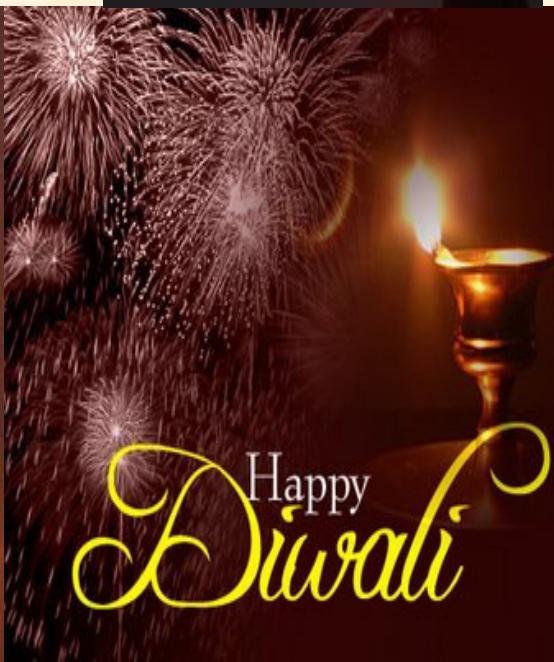
Configuration	Energy (keV)	Coincidence summing correction factor	
		Experimental	Simulated
Soccer-ball	1173.23	0.32 (0.02)	0.33 (0.01)
	1332.50	0.28 (0.02)	0.32 (0.01)
Castle	1173.23	0.63 (0.02)	0.65 (0.06)
	1332.50	0.62 (0.02)	0.63 (0.05)

## Summary

Close geometry efficiency calibration and coincidence summing correction have been performed for a single  $\text{LaBr}_3(\text{Ce})$  cylindrical detector, an array of 32 conical NaI(Tl) detectors in soccer-ball geometry and an array of 14 straight hexagonal NaI(Tl) detectors in castle geometry

A good agreement between simulations and measurements has been achieved

The present work demonstrates the reliability of the coincidence summing correction method for efficiency calibration of 3 very different configurations.



**“Tamasoma Jyotirgamaya”**- From darkness, lead me to light