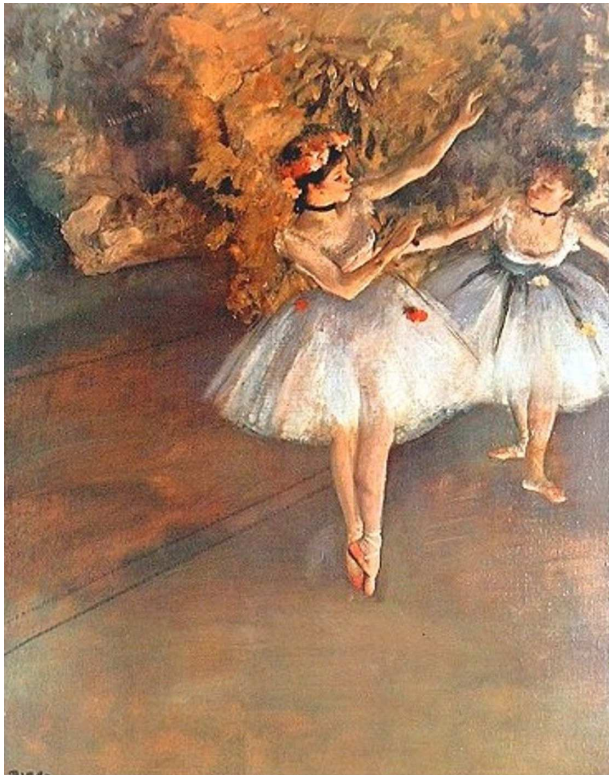
McLaurin, Jacobi and Poincaré shape transitions in rotating nuclei within the optimal shape theory



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PARIS Collaboration Meeting, Bormio, 21 Febbraio 2012

Contents:

Potential energy surface in LDM

The binding energy of a charged and rotating liquid drop is

$$\boxed{E = E_{\rm vol} + E^0_{\rm surf}\, B_S({\rm def}) + E^0_{\rm Coul}\, B_C({\rm def}) + E^0_{\rm rot}\, B_R({\rm def})}$$

The deformation energy (in units $E^0_{\rm surf}$) can be written as:

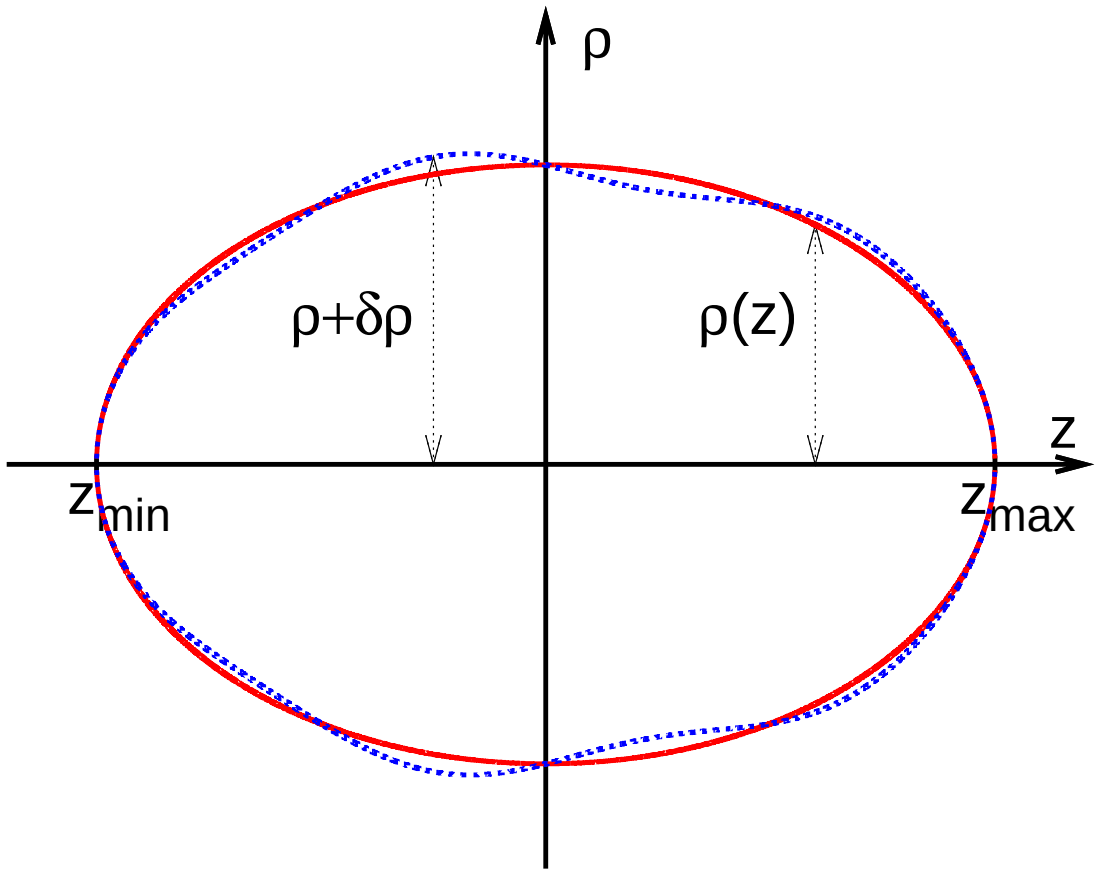
$$\boxed{E_{\rm def} = (B_S - 1) + 2x(B_C - 1) + y(B_R - 1)\ ,}$$

where $x = E^0_{\rm Coul}/2E^0_{\rm surf}$ and $y = E^0_{\rm rot}/E^0_S$ are the fissility and rotational parameters*, respectively.

For the charged drop one has $x > 0$, while $x < 0$ corresponds to gravitating objects.

*S. Cohen, F. Plasil, W. J. Swiatecki, Ann. Phys. **82** (1974) 557.

Strutinsky's variational procedure*:



* V. M. Strutinsky et al. Nucl. Phys. **46**, (1963) 659.

Optimal shapes theory*

Let us consider an axially symmetric shape given by the equation:

$$\boxed{\rho = \rho(z) \qquad \text{for} \qquad z \in [z_{min}, z_{max}] \ ,}$$

where $\rho = \sqrt{x^2 + y^2}$ is the distance from the z-axis to the nuclear surface with the following boundary conditions:

$$\boxed{\rho(z_{min}) = \rho(z_{max}) = 0, \ \left(\frac{d\rho}{dz}\right)_{z_{min}} = \infty, \ \left(\frac{d\rho}{dz}\right)_{z_{max}} = -\infty \ .}$$

The deformation energy of such a body is given by the integral:

$$\boxed{f(\text{shape}) = \int_{z_{min}}^{z_{max}} \xi(\rho, \rho', z) dz \ ,}$$

as shape dependent functions: B_S, B_C and B_R are integrals of this type.

*V. M. Strutinsky et al. Nucl. Phys. **46**, (1963) 659.

The liquid-drop deformation energy , i.e. the functional $\mathbf{f(shape)}$, reaches a local extremum (possibly a minimum) when

$$\boxed{\delta_\rho \mathrm{f} \ = \delta_\rho \int\limits_{\mathrm{z_{min}}}^{\mathrm{z_{max}}} \xi(\rho, \rho', \mathrm{z}) \mathrm{dz} = 0 \ ,}$$

what leads to the Euler-Lagrange equation:

$$\boxed{\frac{\mathrm{d}}{\mathrm{dz}} \frac{\partial \xi}{\partial \rho'} - \frac{\partial \xi}{\partial \rho} = 0 \ ,}$$

which solution gives the so-called optimal shape of the nucleus.

The volume conservation and the deformation of the nucleus are imposed through appropriate constrains for the volume $\mathrm{V} = \mathbf{const}$ and e.g. the quadrupole moment $\mathrm{Q} = \mathbf{const}$:

$$\boxed{\widetilde{\xi}(\rho, \rho', \mathrm{z}) = \xi(\rho, \rho', \mathrm{z}) - \lambda_1 \mathrm{V}(\rho, \mathrm{z}) - \lambda_2 \mathrm{Q}(\rho, \mathrm{z}) \ ,}$$

where λ_i are the corresponding Lagrange multipliers.

Optimal shapes in the nonaxial case *

The variation principle for the deformation energy of the liquid drop with surface given by $\rho = \rho(z, \varphi)$ can be written as

$$\boxed{\delta_\rho \mathrm{f} = \delta_\rho \int_{\mathrm{z_{min}}}^{\mathrm{z_{max}}} \mathrm{dz} \int_0^{2\pi} \mathrm{d}\varphi \; \xi(\rho, \frac{\partial \rho}{\partial \mathrm{z}}, \frac{\partial \rho}{\partial \varphi}; \mathrm{z}, \varphi) = 0\,.}$$

This leads to a generalised Euler-Langevin equation:

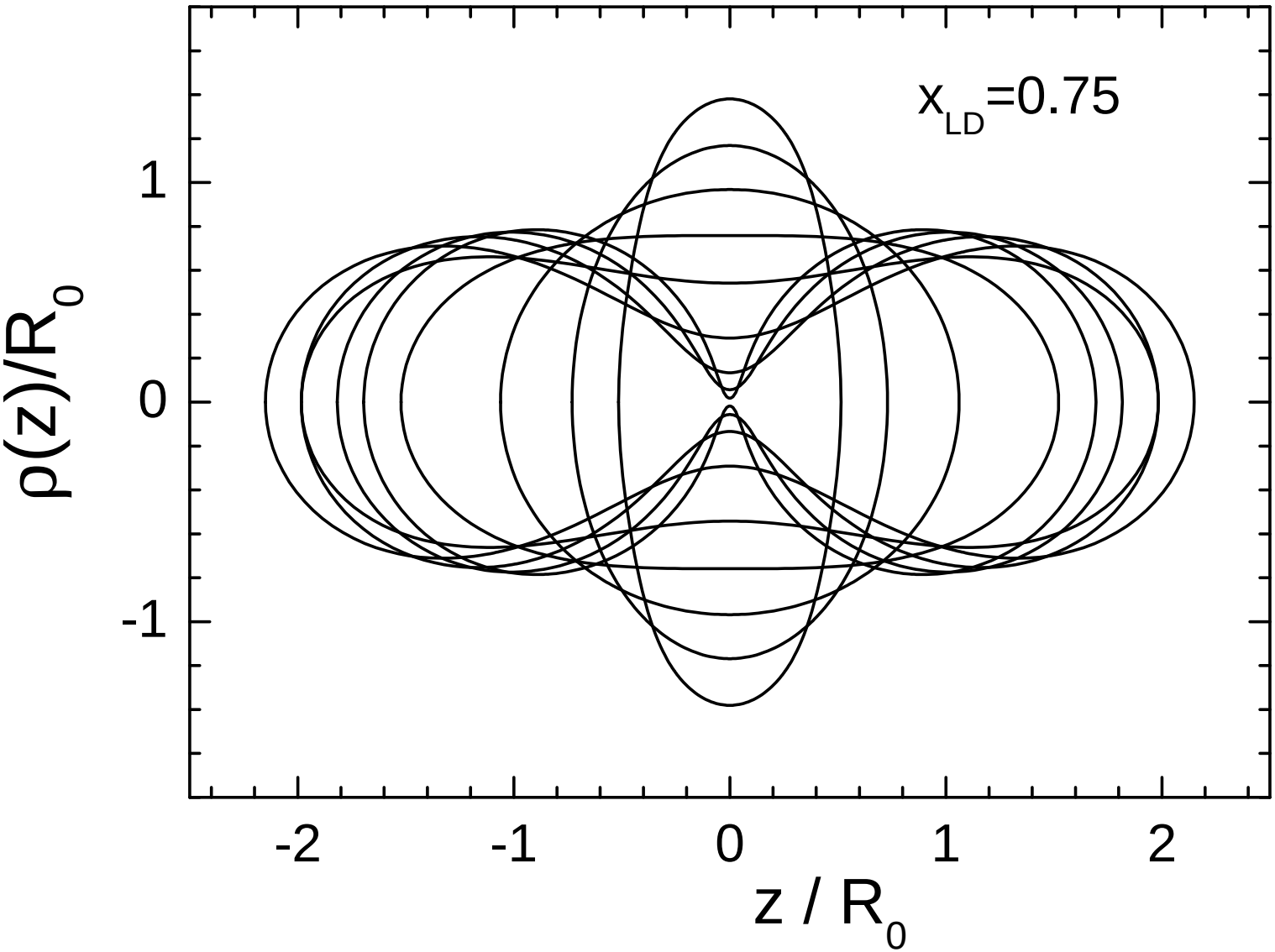
$$\boxed{\frac{\partial}{\partial \mathrm{z}} \frac{\partial \xi}{\partial \rho_{\mathrm{z}}} + \frac{\partial}{\partial \varphi} \frac{\partial \xi}{\partial \rho_\varphi} - \frac{\partial \xi}{\partial \rho} = 0\,,}$$

which can be solved with the following boundary and periodicity conditions:

$$\boxed{\rho|_{\mathrm{z_{min}}} = \rho|_{\mathrm{z_{max}}} = 0, \; \frac{\partial \rho}{\partial \mathrm{z}}|_{\mathrm{z_{min}}} = \infty, \; \frac{\partial \rho}{\partial \mathrm{z}}|_{\mathrm{z_{max}}} = -\infty; \; \rho(\mathrm{z}, \varphi) = \rho(\mathrm{z}, \varphi + 2\pi).}$$

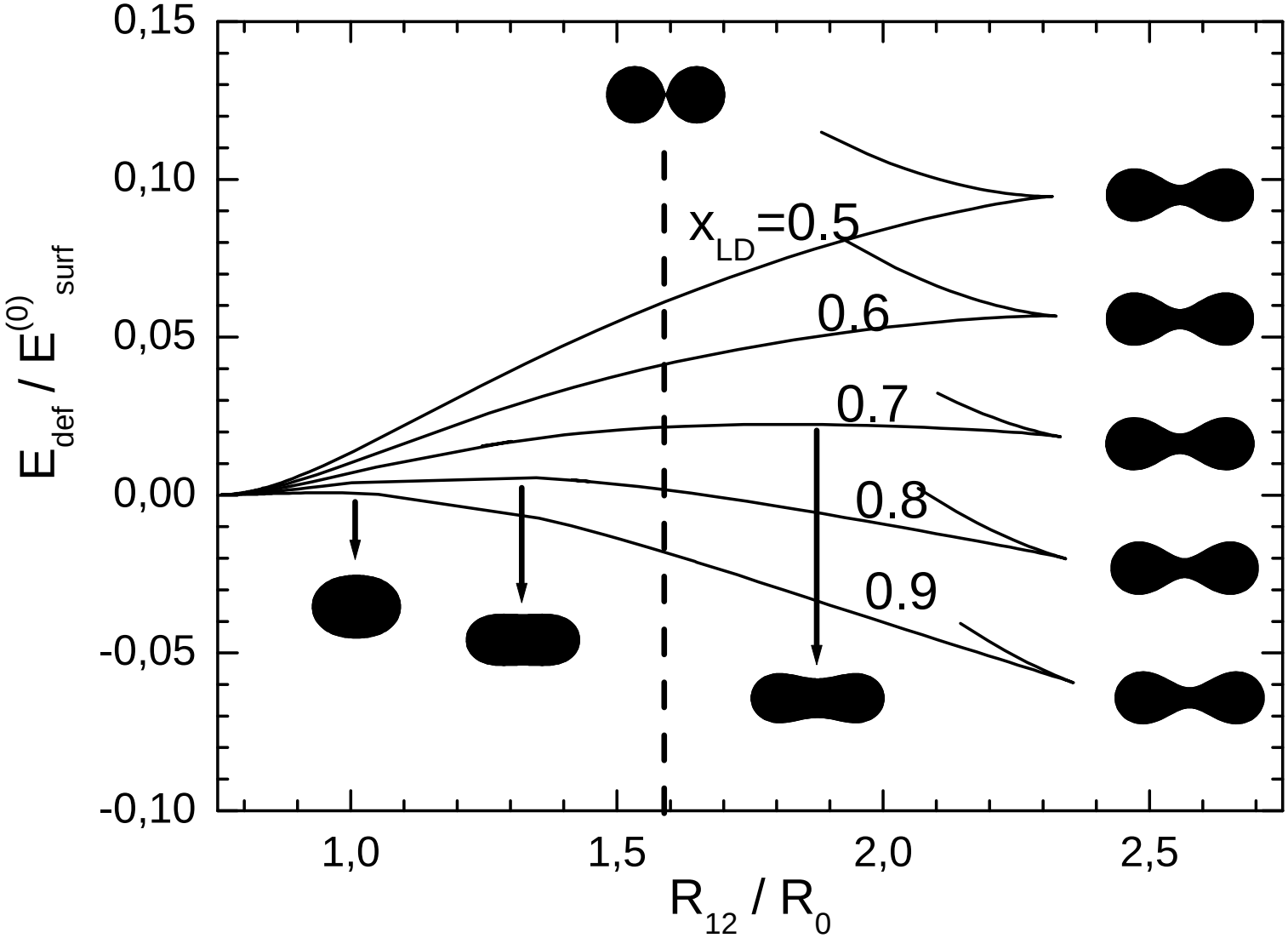
*K. Pomorski, F. Ivanyuk and J. Bartel, Acta Phys. Polon. **B42**, 455 (2011).

Optimal shapes for different elongations*:



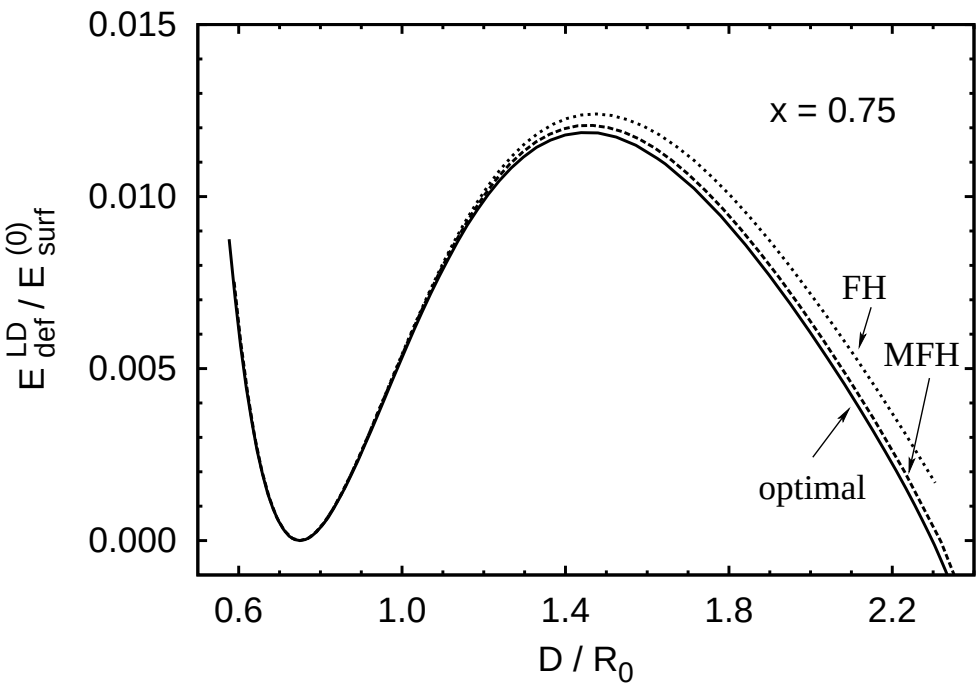
*A. Ivanyuk and K. Pomorski, Phys. Rev. C **79**, (2009) 054327.

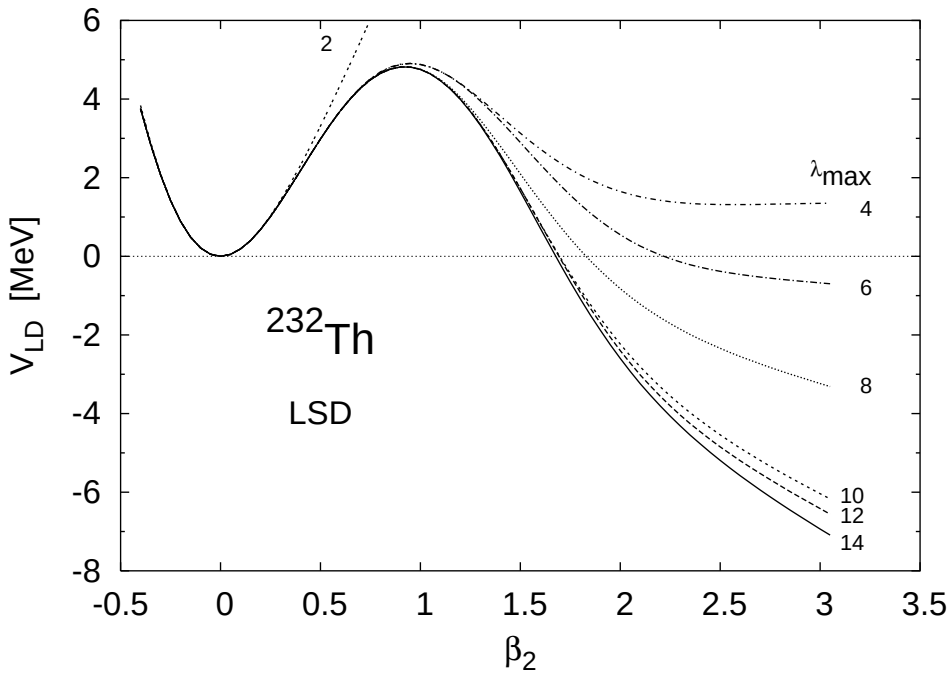
Optimal fission barriers:



A. Ivanyuk and K. Pomorski, Phys. Rev. C **79**, (2009) 054327.

Optimal fission barriers compared with those obtained in different shape parametrisations

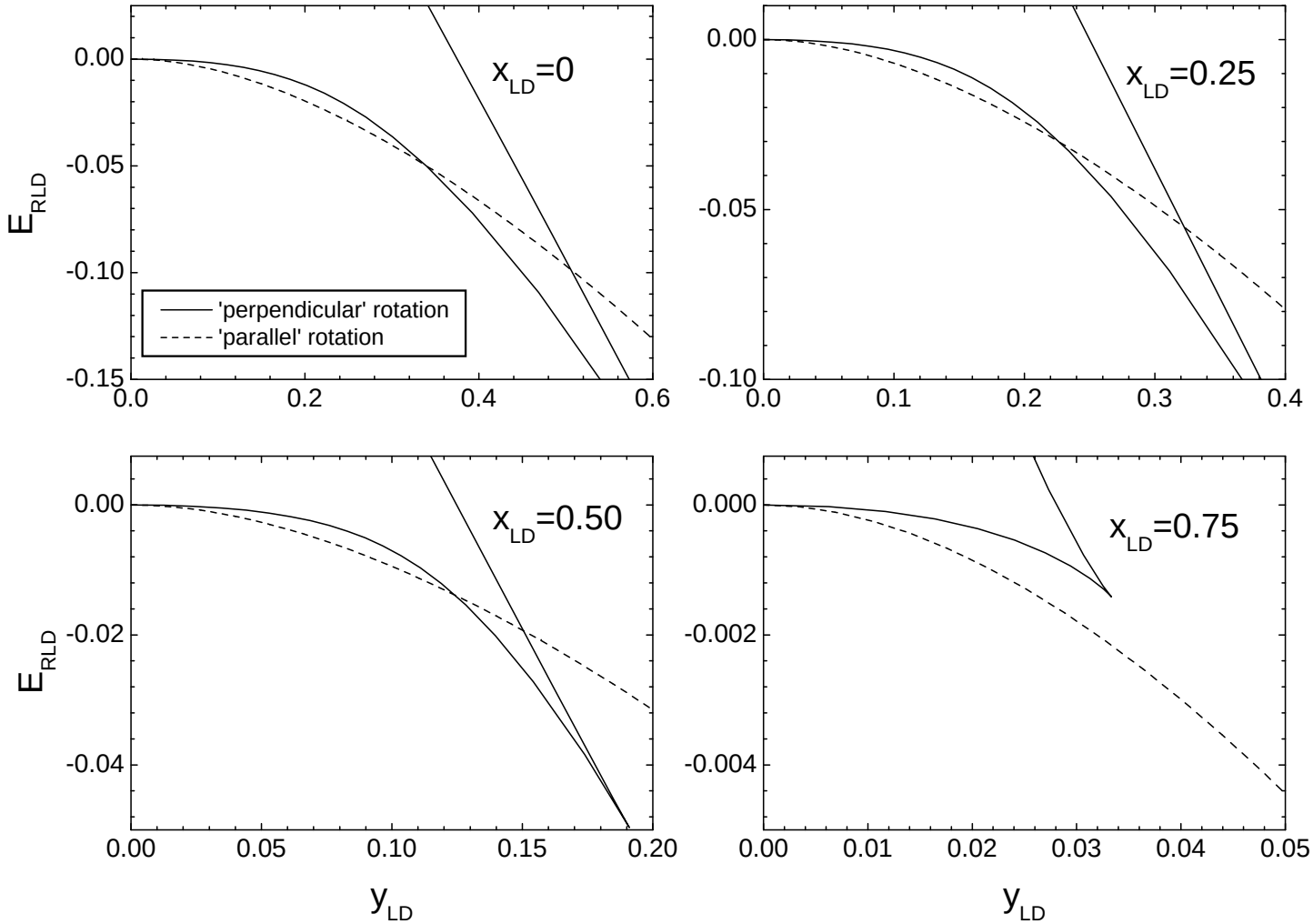




Spherical harmonics expansion

FH – M. Brack et al. Rev. Mod. Phys., **44**, 320 (1972),
MFH – J. Bartel, F. Ivanyuk, K. Pomorski, Int. J. Mod. Phys. **E19**, 601 (2010).
LSD – K. Pomorski and J. Dudek, Phys. Rev. C **67**, 044316 (2003).

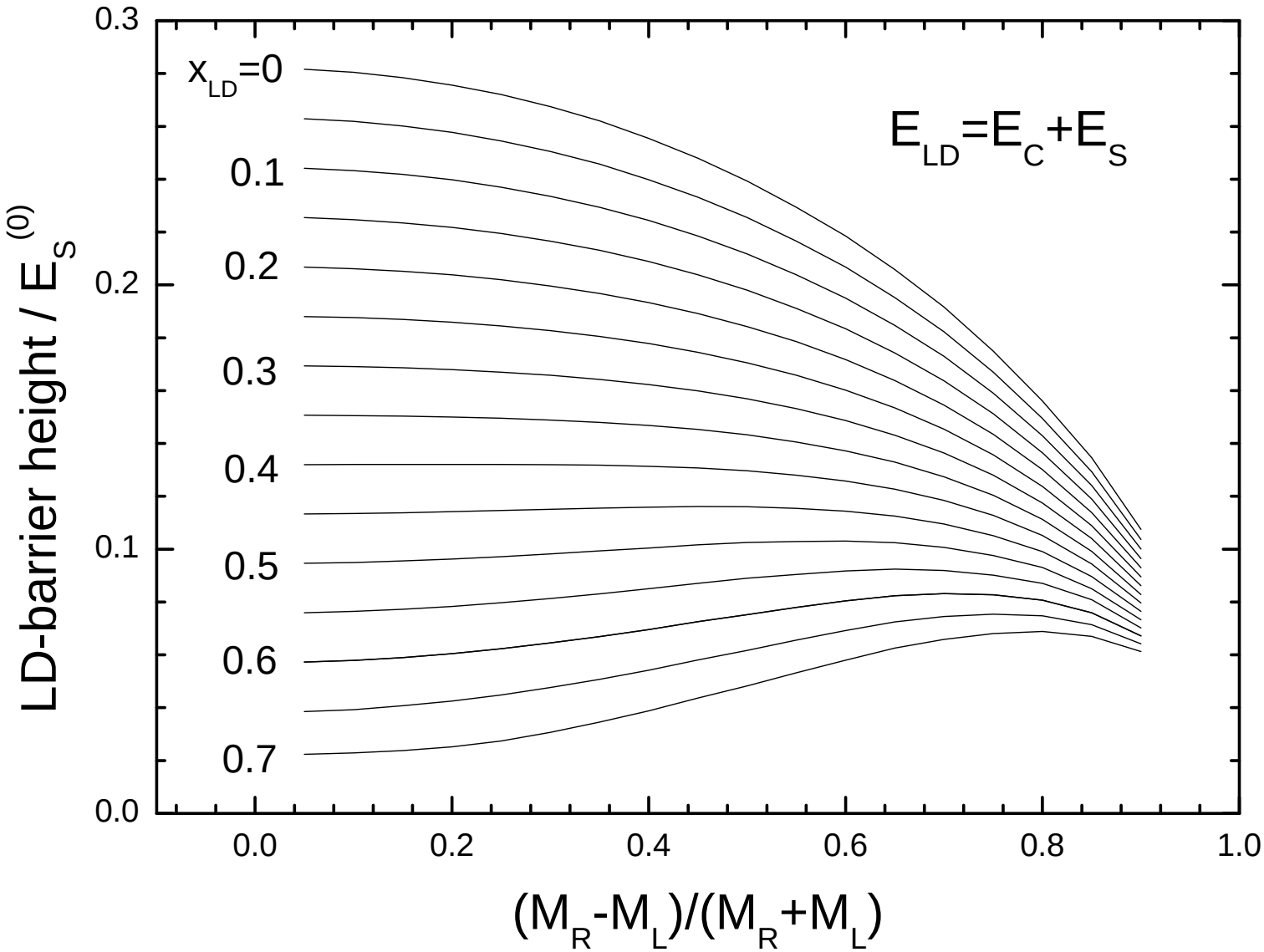
Optimal energy for axially symmetric rotating nuclei *



For $x = 0.5$ oblate to prolate (Jacobi) transition takes here place at $y = 0.125$ what corresponds to E_{rot}=60 MeV and L=75$\hbar$. For $x > 0.612$ no transition is observed.

*J. Bartel, F. Ivanyuk, K. Pomorski, Int. J. Mod. Phys. **E19** (2010) 601.
F. Ivanyuk, K. Pomorski, J. Bartel, Int. J. Mod. Phys. **21** (2012) in print

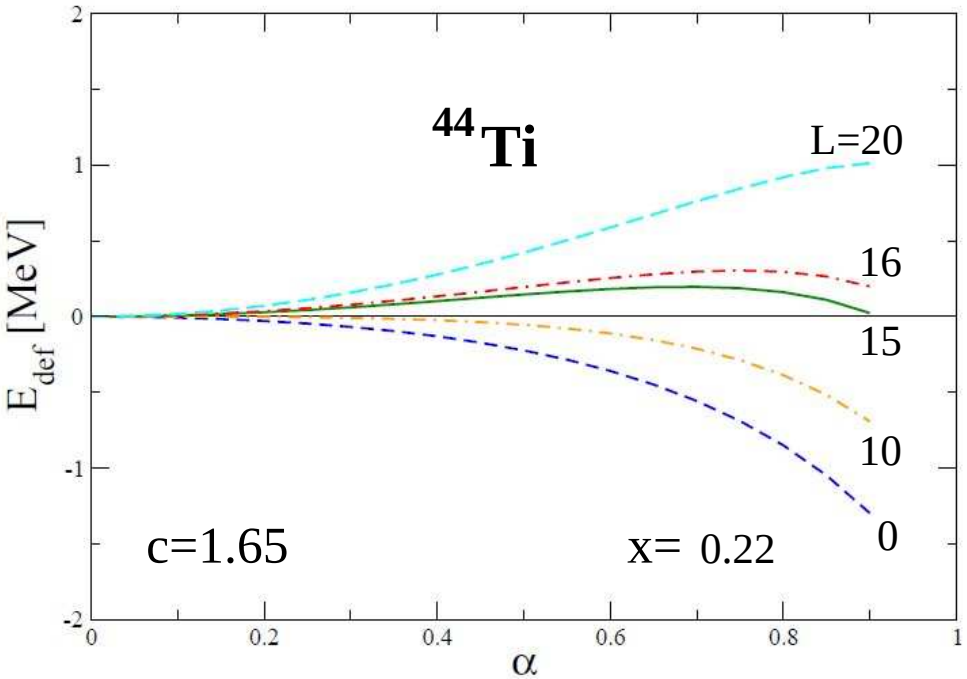
Asymmetry of saddle points for the optimal shapes *

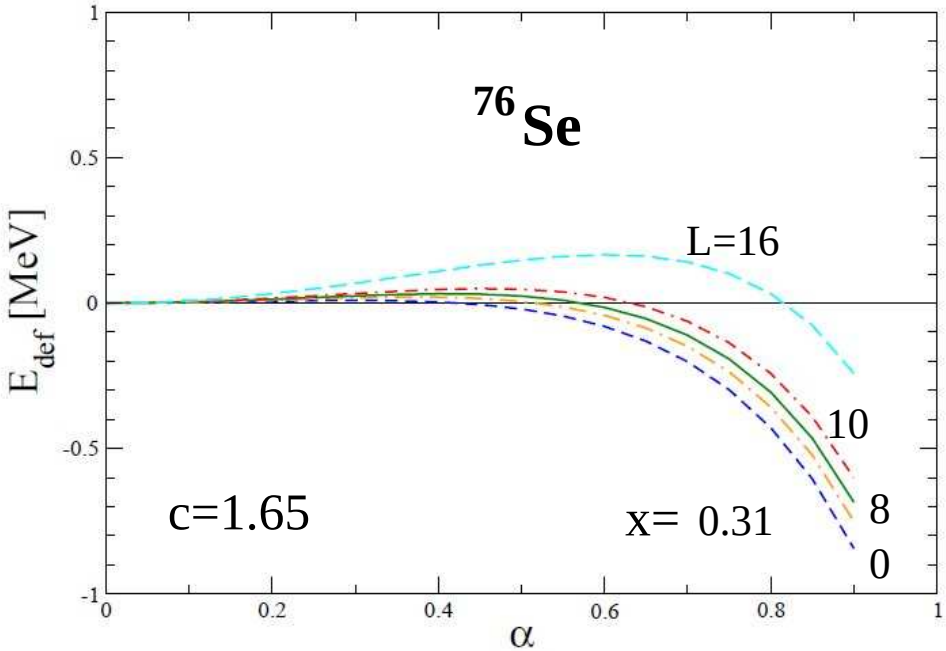


*F. Ivanyuk and K. Pomorski, Int. J. Mod. Phys. **E19** (2010) 514;
see also U.L. Businaro and S. Gallone, Nuovo Cim. **5** (1957) 315.

$x_{\mathrm{BG}} = 0.394$

Restoring of left-right symmetry by rotation

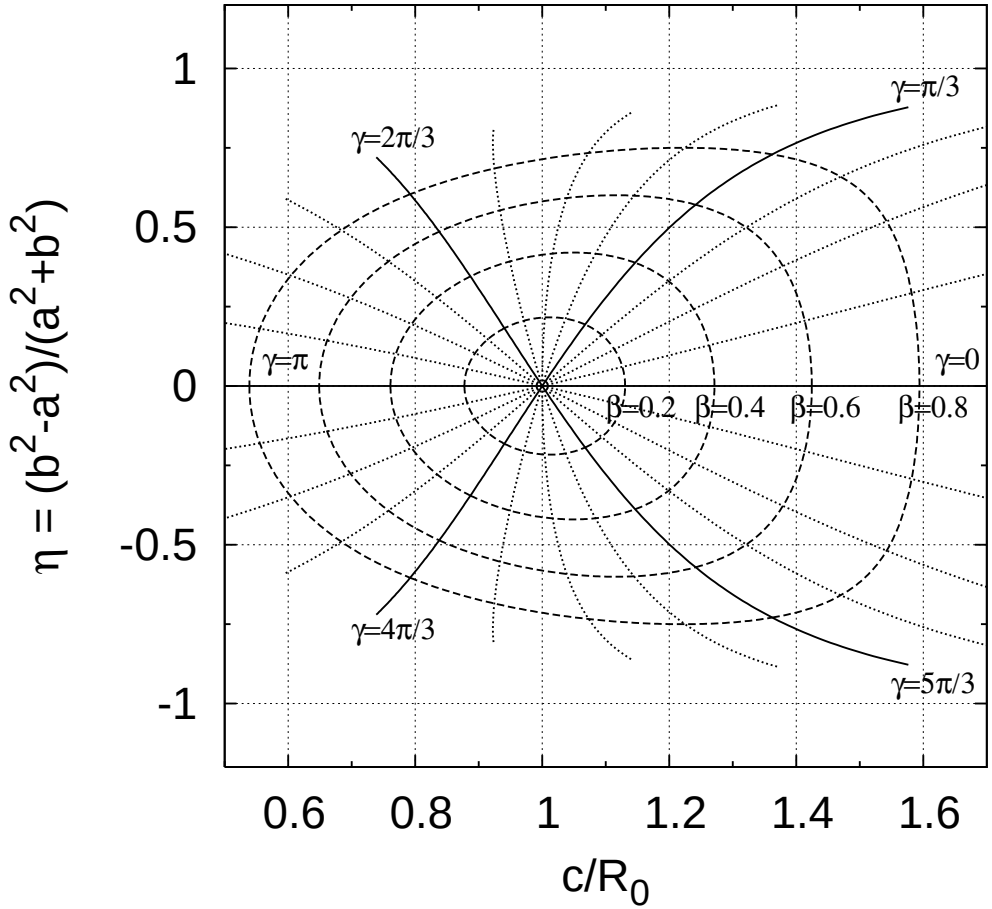




Here, c is the elongation and α is the mass-asymmetry deformation parameter.

J. Bartel and K. Pomorski, Int. J. Mod. Phys. **E20**, 333 (2011).

Nonaxial deformation parameter *



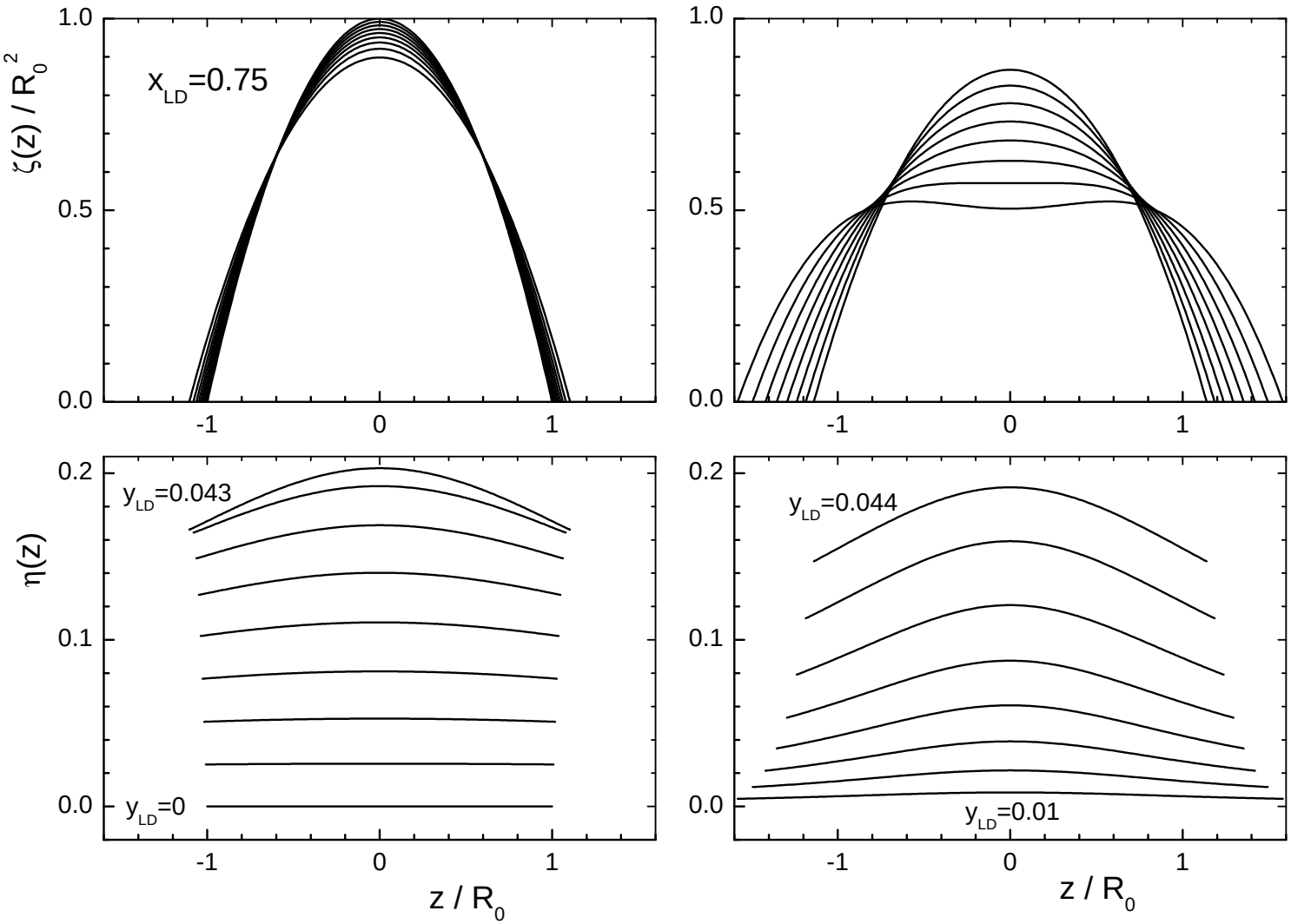
Approximative solution of the generalised Euler equation:

$$\tilde{\rho}^2(z,\varphi) = \frac{\rho(z)\sqrt{1-\eta^2(z)}}{1+\eta(z)\cos(2\varphi)}$$

Relation between (c,η) and (β,γ) for an ellipsoid.

*J. Bartel, B. Nerlo-Pomorska, K. Pomorski, Int. J. Mod. Phys. **E18**, 986 (2009).

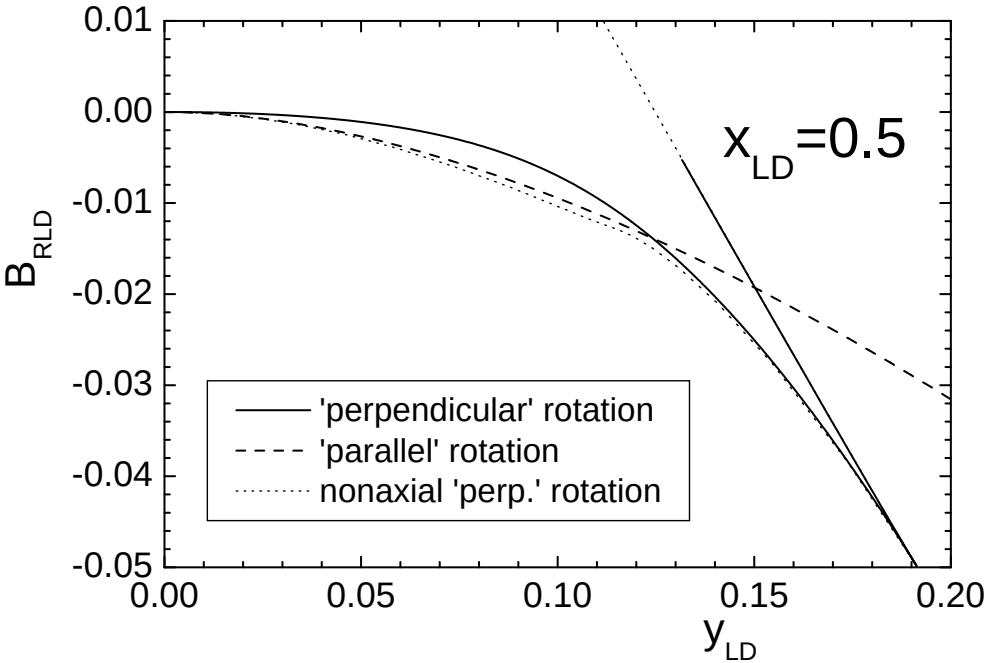
Evolution of nonaxiality with rotation *

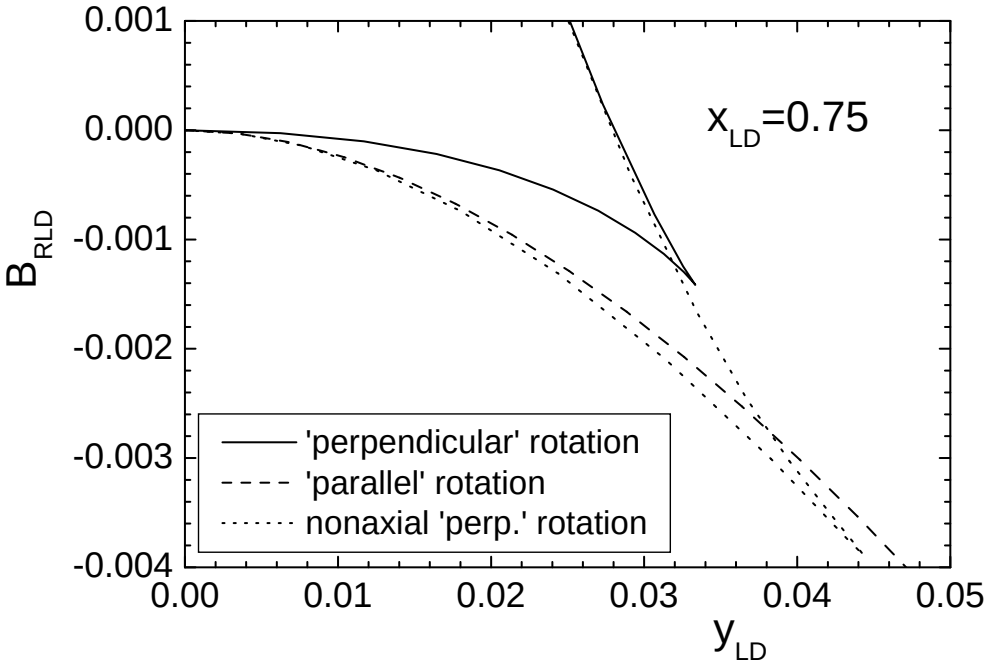


Equilibrium point Saddle point

*F. Ivanyuk, K. Pomorski, J. Bartel, Int. J. Mod. Phys. **E21** (2012), in print

Jacobi transition in rotating drop *



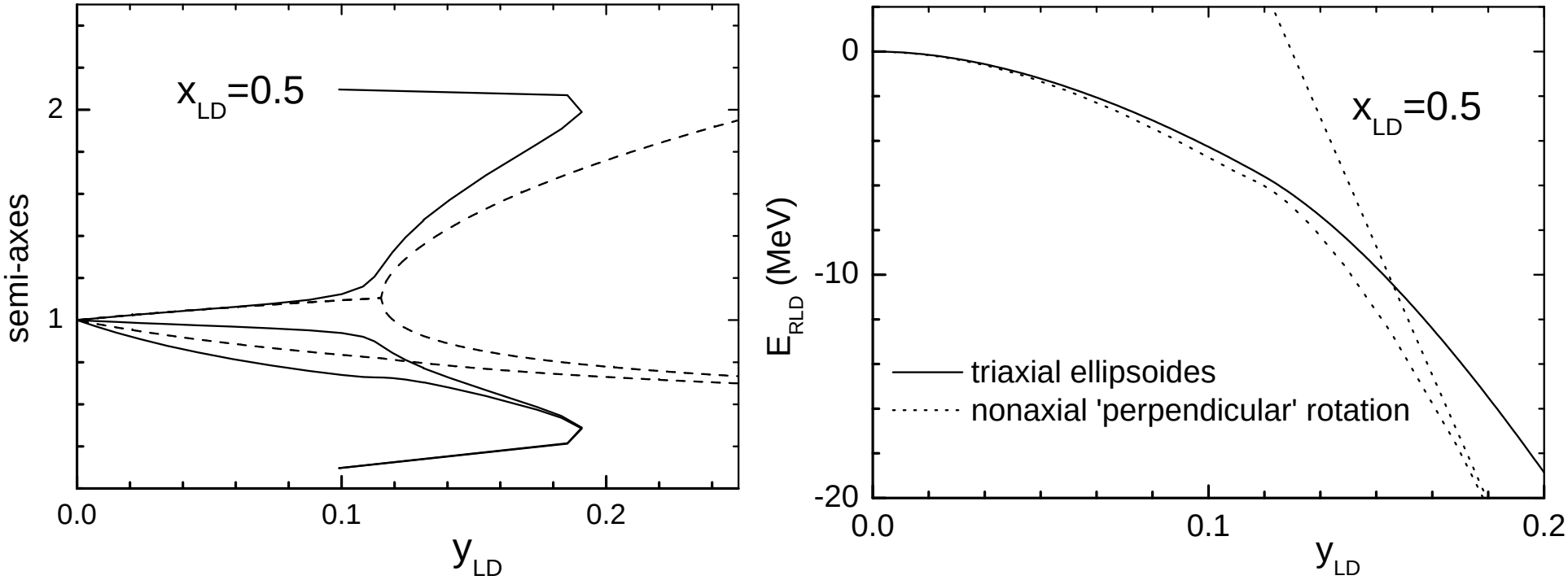


Results for the nonaxial case (dotted line) and $x = 0.5$ (l.h.s. figure) go smoothly from oblate (dashed line) to prolate (solid line) limit.

For $x > 0.612$ (r.h.s. figure) no Jacobi transition is observed.

*F. Ivanyuk, K. Pomorski, J. Bartel, Int. J. Mod. Phys. **E21** (2012), in print

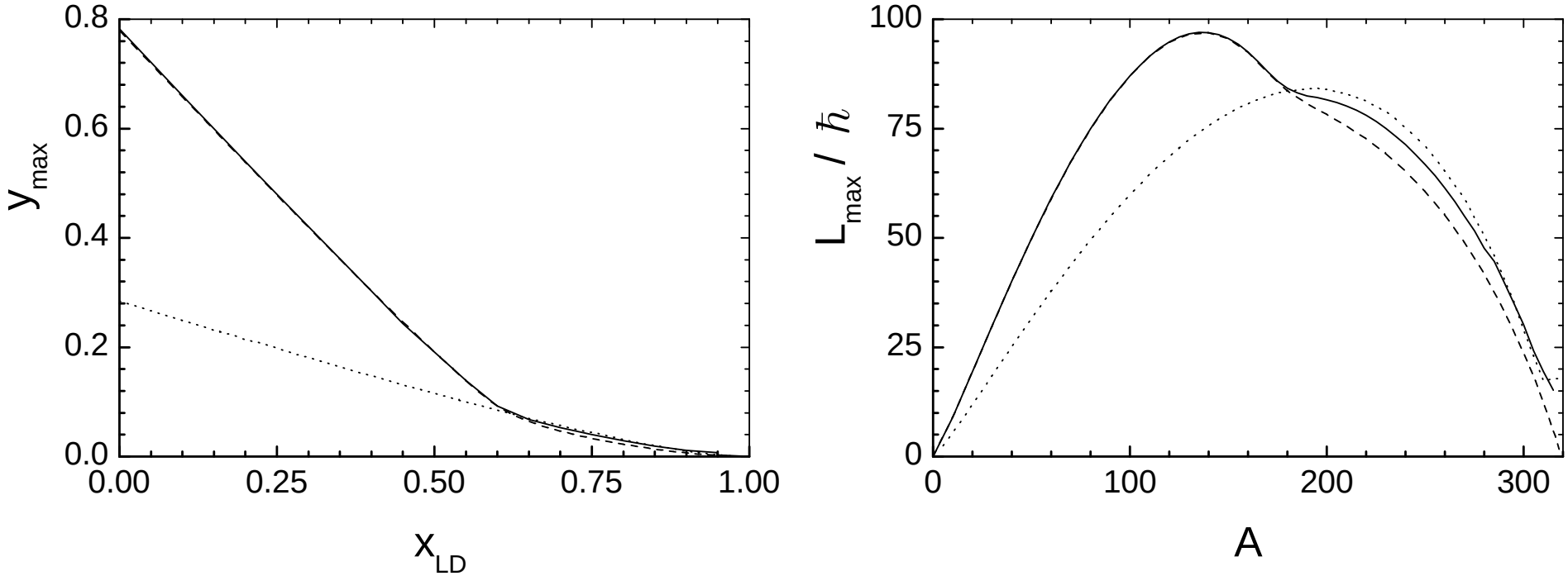
Jacobi transition is not sharp *



Semi-axes of the optimal shapes (solid line) compared with the main $(a,\ b,\ c)$ axes of rotating ellipsoid (l.h.s. figure) in function of the rotational parameter. The optimal solution energy (dotted line, r.h.s. figure) is of course below the energy of rotating ellipsoid (solid line).

*F. Ivanyuk, K. Pomorski, J. Bartel, Int. J. Mod. Phys. **E21** (2012), in print

Maximal angular momentum of rotating nuclei *



Maximal values of the rotational parameter $y_{\mathbf{max}}$ (l.h.s. figure) and of the angular momentum $L_{\mathbf{max}}$ (r.h.s. figure) obtained for the rotation of axially symmetric (solid lines), nonaxial (dashed lines) and ellipsoidal (dotted lines) shapes. The r.h.s. figure is made for the β-stable nuclei.

*F. Ivanyuk, K. Pomorski, J. Bartel, Int. J. Mod. Phys. **E21** (2012), in print

Summary and conclusions:

- Strutinsky's theory of optimal shapes is an useful tool to study the potential energy surface in liquid-drop model.
- The optimal shapes allow to test different parametrisations of the surface of fissioning and rotating nuclei.
- Rotation stabilise nuclei against the Poincaré instability.
- Jacobi transition is not sharp and it disappears completely for nuclei with $x > 0.612$.
- Rotating ellipsoid model underestimates the maximal angular momentum in nuclei with $A < 190$.
- Further calculation are in progress.

Appendix 1: Least-action trajectory

Let us recall that the Hamilton variational principle

$$\boxed{\delta\mathcal{S} = 0}$$

applied to the action $\mathcal{S}$ built from the Lagrange function

$$\boxed{\mathcal{S} = \int_{t_1}^{t_2} \mathcal{L}(q_k, \dot{q}_k, t)dt}$$

leads to the Lagrange equations of motion

$$\boxed{\frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{q}_k} - \frac{\partial\mathcal{L}}{\partial q_k} = 0\,,}$$

which solution $q_k = q_k(t)$ represents the least action trajectory.

Note: All possible trajectories need to have the same beginning and end points, $q_k(t_1)$ and $q_k(t_2)$ respectively and should not differ sustantially form the real trajectory.